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Portrait of Luca Pacioli and Disciple: A New, Mathematical Look

Renzo Baldasso

The *Portrait of Luca Pacioli and Disciple* (Museo e Gallerie Nazionale di Capodimonte, Naples) has been featured in numerous surveys of the history of mathematics, of the history of science, and of the Renaissance (Fig. 1).[1] Yet the only known facts about this painting are that one of its two figures portrays Luca Pacioli and that, according to the *cartellino* (cartouche), it was painted in 1495—the *cartellino* bears the inscription "IACO.B AR.VIGEN/NIS. P. 1495" (Fig. 2). Art historians, intrigued by the stylistic elements, progressive for such a date, have debated its attribution for over a century. Historians of science have tried to identify the puzzling mathematical references and objects that are prominently displayed in the panel and that make it a favorite illustration for surveys of early modern science. The main difficulty in interpreting this painting, however, is to understand it as a whole, identifying its subject and meaning as explicated by its pictorial, iconographic, and cultural elements and their interrelations. As I will contend, this painting portrays not just two individuals but rather the subject of mathematics as a mode of thinking, as a court activity, and as a form of education imparting the new visual literacy that emerged from the scientific studies of humanists during the second half of the fifteenth century.[2] The various mathematical objects displayed are not merely attributes that qualify Pacioli as teacher or elements for a visual biography of the friar, they are part of the panel's exposition of the new mathematics as an abstract and novel subject. This exposition—which, I believe, was invented by Pacioli himself—aims at making the mode of thinking supported by this new mathematics important to the viewers, directly challenging them to engage it instead of simply contemplating the picture and its subject. It is for this purpose that the painter offered to the beholder a pen with which to write and a square and a compass with which to draw geometric diagrams. The focal point of this painting, these geometric figures formed the basis for the grammar of the visual analytic reasoning that, as a form of knowing, was being introduced into court culture during the second half of the fifteenth century by Pacioli and other mathematical humanists.[3]

The Painting

Since 1903 this panel has been part of the collections of the Museo di Capodimonte in Naples; unfortunately, its early provenance is incomplete. Our information about its ownership until the nineteenth century derives from three texts. Two seventeenth-century inventories unequivocally refer to our panel, documenting its transfer from the palace of Urbino to the *guardaroba* (personal possessions) of the duchess of Tuscany, Vittoria della Rovere.[4] The earliest reference is by Bernardino Baldi, the Urbinate historian, humanist, renowned mathematician, and author of a biographical history of mathematics (written in 1587–89). Although he gives the most detailed description of the painting, he complicates its identification by suggesting the existence in Urbino of another portrait of the friar. Specifically, in his biography of Pacioli, Baldi praises a portrait by Piero della Francesca:

> In the wardrobe of our most serene princes of Urbino is conserved, by the hand of Pietro de Franceschi, Pacioli's countryman and excellent painter and perspective expert, a naturalistic portrait of this friar, his book, the *Summa aritmetica,* in front of him and several fictive crystals in the form of regular solids suspended from up high; from the lines, reflections, and shadows of these crystals one discovers how accomplished a painter Piero was.[5]

As an expert mathematician, Baldi would have been familiar with both the texts and the editions of the books depicted in the Capodimonte panel, and he would not have confused Pacioli's *Summa*—which is the short title of *Summa de arithmetica geometria proportioni et proportionalita* (1494)—with Euclid's *Elements,* the book that lies open in front of the friar in the panel; nor would he have missed or misread the *cartellino,* which excludes, as it reads now, Piero's authorship. The elements Baldi includes in his account and those he ignores—such as the second figure—indicate that he is describing a different painting: a portrait of Pacioli represented alone.[6] Together with the information from the other two early modern sources, Baldi's description implies that there were two paintings featuring the friar, and that the Capodimonte panel likely derives from a portrait of Pacioli by Piero that is now lost.[7]

From this inference it follows that "Iaco.Bar.," the painter as named by the *cartellino,* reworked the model, adding the second figure, opening Euclid's *Elements* in front of Pacioli, and moving the *Summa* to the side.[8] A careful inspection of the painting supports the hypothesis that the extant painting is based on an original portraying Pacioli alone. For example, pentimenti in the friar's left hand and right wrist may reveal the challenges of modifying their positions (Fig. 3).[9] Moreover, although conceived to provide the beholder ready visual access to the objects on the table, the awkward perspective rendering of the *Elements* demonstrates the painter's difficulty in locating precisely the vanishing point of the original painting.[10] Also noteworthy is the peculiar situation created by the introduction of the second book and the slate tablet: it left insufficient space to accommodate the entire *Summa,* effectively marginalizing Pacioli's magnum opus, his only printed work in 1495.[11] Finally, the unclear spatial relation of the two figures—which also assigns them a different source of light for which no shadow is cast on the gentleman's glove—offers additional evidence for considering this painting as dependent on and reworking a portrait featuring Pacioli with his book in a balanced and fully symmetrical

1 Jaco.Bar., *Portrait of Luca Pacioli and Disciple,* 1495, oil on panel, 38⅝ × 42½ in. (98 × 108 cm). Museo e Gallerie Nazionale di Capodimonte, Naples (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

composition.[12] Certainly, the sum of these infelicities makes it difficult to imagine that Baldi would not only ascribe the Capodimonte panel to Piero but also actually characterize it as an eminent example of the painter's art and a demonstration of his knowledge of perspective.[13]

Regarding the painting's authorship, radiographic and reflectographic investigations have confirmed the authenticity of the *cartellino,* resolving previous controversy.[14] Reading it as "Jacopo de' Barbari Vigennis Pinxit 1495," current opinion assigns the panel to the hand of this artist.[15] Active in Venice before 1500 and in northern courts afterward, Jacopo de' Barbari is famous above all for his monumental bird's-eye view of Venice, a woodcut composed of six blocks and printed in 1500, and for having been the first Italian artist to produce (what we would now call) a still life painting.[16] These two achievements resonate well with the contents of the Capodimonte panel, the first for the mathematical and perspective demonstration, the second for the attention to naturalistic representation and the unprecedented subject. Even though this would then be Jacopo's earliest surviving work, retrospectively the attribution is plausible on stylistic grounds: like this panel, his later oeuvre reveals an exceptional talent for depicting lifelike details, mastery of perspective, interest in appropriating pictorial solutions from northern portraiture, and a distinctive Venetian training that suggests close study of the manners of Giovanni Bellini and Alvise Vivarini.[17]

Paradoxically, the only reservation about this attribution comes from the *cartellino* itself: the specification "Vigennis" (twenty years old) seems at odds with the information found in official documents concerning the pension Jacopo was given in 1512 for "weakness and old age [*débilitation et vieillesse*]." However, as Creighton Gilbert argued, coeval conventions in legal writings and approximate perception of age make it possible that Jacopo was twenty (or twenty-some)

2 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the *cartellino* (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

years old in 1495 when he painted this panel.[18] In favor of an attribution to a young painter stands the fact that this is a high-quality work that also contains pentimenti and slight infelicities, exposing the inexperience of an otherwise talented artist. In short, the available evidence supports Jacopo de' Barbari's authorship. Yet, more interesting than the artist's name would be to know the training and professional background that prepared him to carry out a commission that was both unprecedented in subject matter—thus without an established iconography and artistic tradition—and truly difficult to execute given its considerable pictorial challenges (even if he had access to the hypothetical portrait of Pacioli by Piero and received guidance from the friar about the iconography).[19] Nonetheless, for the history of early modern visual culture and the present enterprise, it is more important to understand the contents of the painting, its raison d'être in 1495, and the appeal to a late-fifteenth-century audience of a seemingly abstruse picture such as this, with mathematical humanism and visual reasoning as subjects.

3 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the friar's right hand (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

The key for confronting these questions is Luca Pacioli himself and the book opened in front of him (Figs. 1, 4). As already mentioned, of the two figures in the painting, his name is the only one known securely. His identification is confirmed by both internal and external evidence. The inscription on the side of the book in the lower right reads "LI R. LVC. BVR."; interpreted as "Liber Reverendi Lucae Burgensis," it identifies the book's author as Luca Pacioli of Borgo S. Sepolcro (Fig. 5). The similarities between the panel's portrayal and the portrait seen in the three L initials in the *Summa* provide independent confirmation (Fig. 6).[20] Specifically, both versions present the friar with his head turned, eyes cast into the distance, and a tense facial expression that defines a moment of reflection, underscoring that he is thinking about abstract problems. Moreover, both are portraits of a real individual rather than standardized represen-

4 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the open book (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

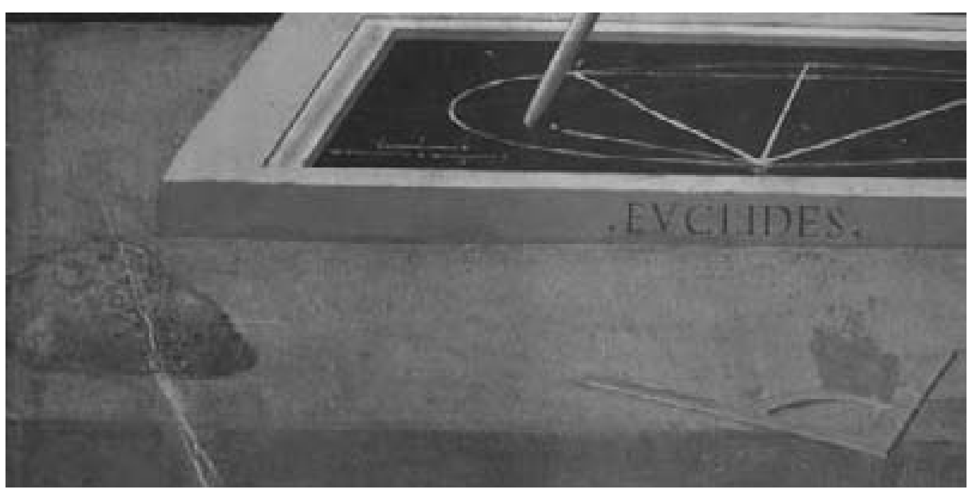
5 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the closed book and dodecahedron (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

tations of the established prototype of the mathematician or *erudito* author.[21] The fact that the images of Pacioli inside the L initial woodcuts and on the panel share important elements lends support to the hypothesis that both renditions depend on a preexisting one—probably Piero's painting—that authoritatively established the "iconography" of the friar as a polymath who thought with and about geometric diagrams.

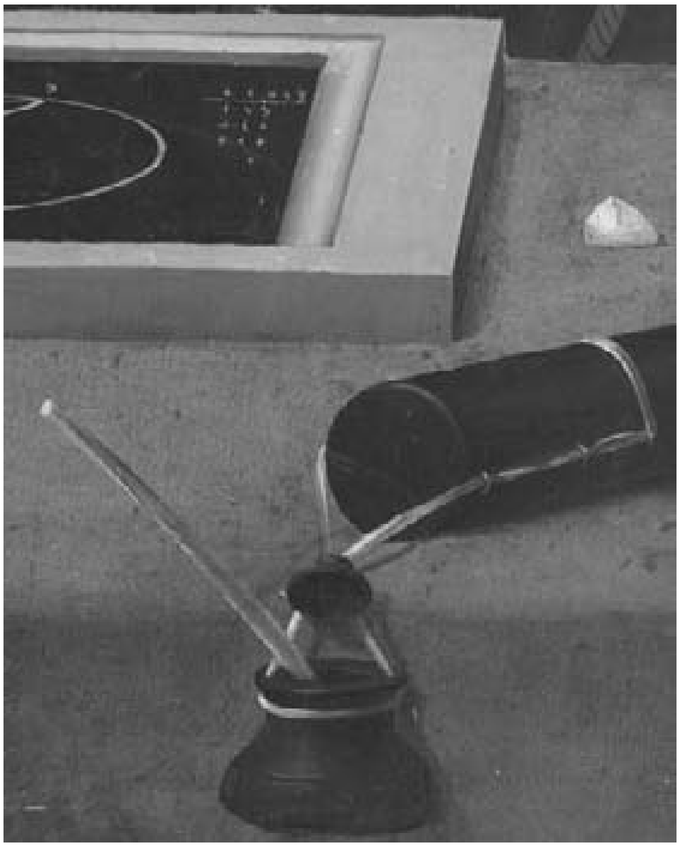
Because the second figure plays an important role in the audience's interaction with the picture and his presence and guise contribute to the legitimation of its contents, the identification of the nobleman would be very useful for better assessing the painting. Unfortunately, no existing documentary evidence has come forth to ascertain his name. Scholars have proposed several candidates, including Guidobaldo da Montefeltro, duke of Urbino, and Albrecht Dürer, each subtending different interpretations for the panel's original function.[22] If the second figure were this or another artist—such as Jacopo de' Barbari himself—the painting could have served as a record certifying a young painter's intellectual relationship with an eminent mathematician (and expert on perspective).[23] If the second figure were the duke, this painting should then be considered a patronage item, possibly commissioned to make official a new relationship that established Guidobaldo, the dedicatee of the *Summa,* as Pacioli's patron, rather than the pupil he had been in his youth when he was tutored by the friar.[24] The visual evidence indicates this latter as the more probable case: the figure's height and slender physique make this young nobleman appear not only physically taller than the friar but also as somebody with a higher social standing, a position suitable above all to a patron. His countenance and fancy garments give him the authority to address and guide the viewer. Finally, the different light that illuminates his forearm becomes meaningful, suggesting a definite intellectual and cultural enlightenment.[25]

Concerning the various mathematical objects and references in the painting, the most detailed and convincing description has been presented by Enrico Gamba; however, even he has admitted to have solved only pieces of the puzzle.[26] The friar's left index finger indicates the start of Proposition 8 of book 13 in Euclid's *Elements.* The stick that Pacioli holds in his right hand points instead to an interrupted line of unclear meaning in a figure that is related to the one visible next to the passage (Proposition 8) the friar fingers in the book (Fig. 7). On the slate, together with some lines denoting proportional relations, are three numbers that are correctly summed—Gamba related two of them to the figure (Fig. 8).[27] On the painting's right side lies the *Summa,* on which stands a wooden dodecahedron, a perfect or Platonic solid (Fig. 5). Finally, on the picture's upper left, a rhombicuboctahedron, a semiperfect or Archimedean solid, hangs from a wire attached to its bottom face (Fig. 9).[28] The translucent solid, half filled with what may be water, carries the same image of a princely mansion, probably intended to be Urbino's palace, sketchily reflected three times; it also contains the reflection of the green tablecloth together with Pacioli's lighter and the nobleman's darker presence.[29] On the table is an eraser, an unruled try square, an empty pen container to which is attached an inkwell that hangs in front of the table with a quill in it, a small piece of chalk, and a compass. Although it is evident that these objects are instruments for writing and drawing geometric figures on slate and paper, the significance of their prominence and placement has not been explored in the scholarly literature. It is noteworthy that save for the chalk, these writing and drawing instruments are not within easy reach of either Pacioli or the gentleman next to him; they are placed closer to the viewer.[30] The orientation of the compass and the square together with the pen's position clearly indicate that these instruments are offered to the beholder: they are ours to write and draw with. Before discussing further the presentation and meaning of this ensemble of instruments and mathematical references, it is important to consider Pacioli in his role as a leading

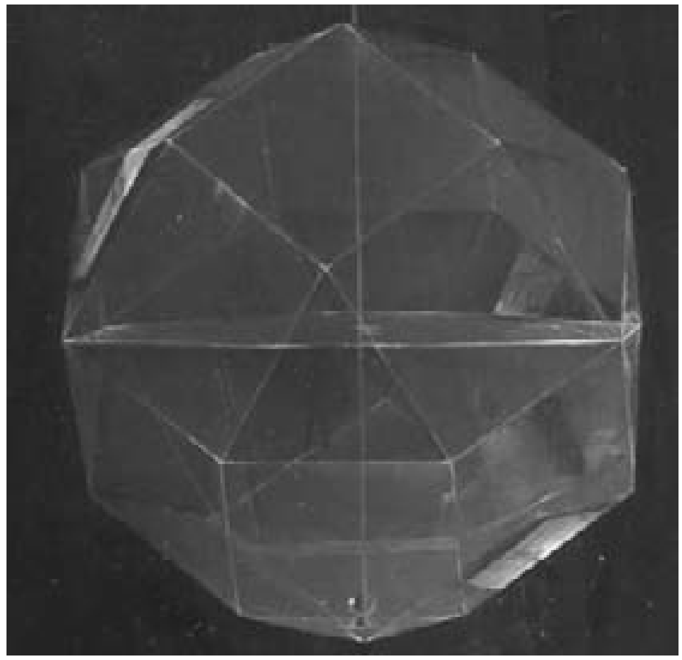
6 Text opening with L initial from Luca Pacioli, *Summa de arithmetica geometria proportioni et proportionalita,* Venice: Paganinus de Paganinis, 1494, 1r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (artwork in the public domain; photograph by the author)



7 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the slate tablet (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)



8 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the right side of the slate and the pen (artwork in the public domain; photograph by the author, reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)



9 Jaco.Bar., *Luca Pacioli and Disciple,* detail of the crystal polyhedron (artwork in the public domain; reproduced by courtesy of the Sopraintendenza of the P.S.A.E. and Polo Museale della Città di Napoli)

protagonist in late-fifteenth-century mathematical humanism and in the history of the affirmation of visual reasoning in Renaissance intellectual culture.[31] He is, self-evidently, a crucial figure for the interpretation of the panel.

Luca Pacioli

In his biographical account of Piero della Francesca, Giorgio Vasari describes Pacioli in very negative terms: as one of those "who seek to conceal their asses' skin under the honorable spoils of the lion," that is, one of those who hide their ignorance by stealing the fruits of the studies completed by true intellectuals.[32] Well established already in the early modern period, this charge of plagiarism rendered this mathematician an unattractive subject for research; until very recently, there was no sustained study of Pacioli's extensive mathematical writings.[33] Argante Ciocci's monograph *Luca Pacioli e la matematizzazione del sapere nel Rinascimento* (2003) now offers a starting point for considering this polymath's complicated career and influential works, which intersected with Renaissance courts, artists, princes, university teaching, as well as ancient, medieval, and early modern authors.[34] Here I will limit my considerations to the elements necessary to understand the contents of the panel.

For Pacioli's formation, as for most protagonists in the history of visual reasoning in Renaissance science, the immersion in the Venetian educational and intellectual contexts of the last third of the fifteenth century proved crucial.[35] He first arrived in Venice in 1464, having mastered the basics of accounting and mathematics in his native S. Sepolcro (possibly under Piero's guidance, as Vasari suggested).[36] In the Venice of Ermolao Barbaro and Johannes of Speyer, Pacioli lived with the Rompiasi, a wealthy family of merchants, as their accountant as well as tutor and study mate of their sons.[37] As he recounts in an autobiographical note in the *Summa,* during the next several years he attended the lectures of Domenico Bragadino at the Scuola di Rialto, receiving an advanced education in mathematics and natural philosophy as interpreted by the Venetian Aristotelian tradition.[38] Perhaps attracted by the prominence that mathematical studies had gained at the papal court, in 1471 Pacioli moved to Rome, where he lived in the house of Leon Battista Alberti.[39] Sometime following Alberti's death in April 1472 and before 1477, Pacioli, who was born into a modest family, took the vow of poverty in the Franciscan Order, seeking the means to pursue his researches and to gain access to places of high culture. Although mathematics remained Pacioli's principal interest, he also studied theology, thereby acquiring the credentials that allowed him to travel widely and to find academic employment. Over the course of his long career—which brought him to Bologna, Florence, Milan, Naples, Rome, and Urbino—he had occasion to interact with scholars, patrons, and artists, including, among others, Leonardo da Vinci and Albrecht Dürer; he also held teaching appointments (always in mathematics) at the *studia* (universities) of Perugia, Rome, Naples, Milan, Pavia, and Florence.[40] These residencies and lectureships indicate that his ability to bridge knowledgeably philosophy, theology, and mathematics, while addressing theoretical and practical issues, made the friar a sought-after intellectual and courtier.

Witness to the style of the Capodimonte panel, Venetian connections shaped Pacioli's endeavors throughout his many moves, as well as his publications, which were all printed in Paganino Paganini's Venetian print shop. The best evidence for the friar's special relationship to the Serenissima was the crowd of Venetians that attended the opening lecture (focused on book 5 of the *Elements* and published in the printed edition of Pacioli's *Divina proportione*) of his course on Euclid in the summer of 1508: over five hundred attendees filled the church of S. Bartolomeo to capacity on August 11, 1508.[41] Evidently, Venetians appreciated the importance of Pacioli's researches as well as his ability to digest a wealth of information from a multitude of sources and to make his erudition interesting to an audience of nonspecialists.

To understand the selection of elements that the panel presents to represent Pacioli, it is useful to briefly consider what is absent. As this portrait remains oblivious to many of the friar's achievements and interests pursued during a life dedicated to research and teaching, the painting cannot be considered a visual summary of Pacioli. Above all, there is no reference to abacus mathematics, the bread and butter of his teaching and a subject to which he dedicated considerable attention in his writings.[42] Moreover, the panel does not celebrate Pacioli as teacher: the scene is set in a *studiolo*-like space instead of a lecture hall—standing at once next to and behind Pacioli, the nobleman cannot even see what the friar is pointing out on the tablet, and so he should not be considered a student or tutee. Nor does the panel celebrate the Franciscan as author: his own book is closed and set aside. Omitting these otherwise defining dimensions of Pacioli, the panel presents the friar as a master of Euclidean geometry and a mathematical humanist, a portrayal supported in 1495 in print only by the *Summa*'s concluding section, which offers a compendium of the *Elements*. These seventy-six leaves constitute only a prelude to his labors on this text, which would occupy him for most of his career and of which he produced a Latin edition (eventually published in 1509) and an Italian translation (unfortunately lost).[43] But more than an expert on the *Elements,* the friar is here shown as an authority on its figures and a master at reasoning through geometric diagrams, both in two-dimensional constructions and in three-dimensional solids. To convey this point visually, Pacioli is depicted considering figures from book 13 of Euclid's *Elements*.

The experiences and studies that prepared Pacioli for this position were probably various, and it is difficult to point to a particular text or person. One can imagine that he studied one of the mathematical manuscripts filled with new geometric figures copied in Rome in the late 1450s; one can also suppose that he studied works by Piero, Alberti, or Nicholas of Cusa, contemporary authors who made conspicuous use of mathematical diagrams in their writings.[44] Yet it should be stressed that Pacioli himself was a leading proponent of reasoning by visual means, specifically through Euclidean diagrams. The figure on the tablet displays his commitment to the affirmation of a visual analysis that is based on geometric forms and rules, while the diagrams in the book feature the first and famous *printed* instances of such figures and confirm his affiliation with the avant-garde of mathematical humanism. These geometric figures and printed diagrams were novelties both as theoretical subjects and as material artifacts: they were new analytic tools and typographic innovations, as well as concrete instances of the later-fifteenth-century efforts to recover the visual dimension of ancient mathematical and scientific researches, a dimension that medieval scribes had obscured or ignored. Certainly, the hundreds of figures and diagrams that Pacioli included in his treatises—the first modern mathematical works to be printed—testify to his interest in promoting the employment of geometric figures and graphic means more generally in mathematical studies, as well as in other situations where an analytic approach was welcome.[45] These details and iconographic choices pose a critical challenge to the interpretation of this painting as a double portrait set as a teaching scene and encourage a more careful analysis of the picture's contents and of Pacioli's presentation.

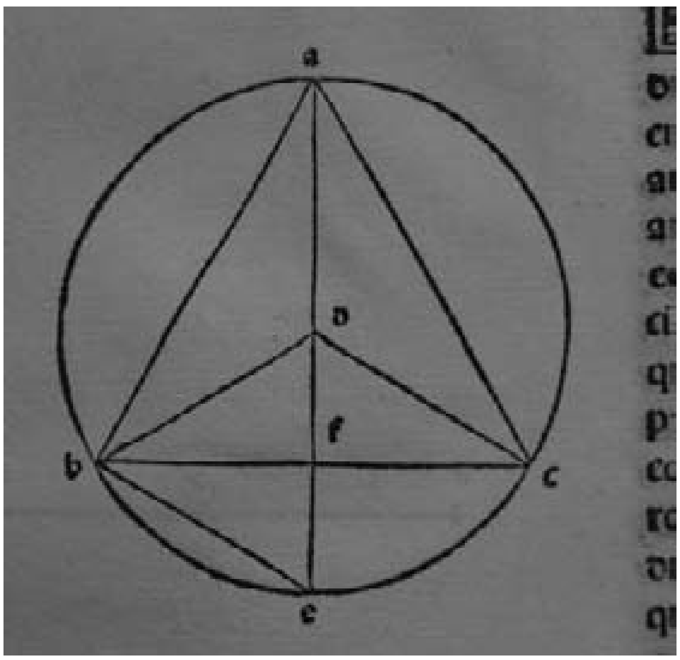
Painting Pacioli / Picturing Mathematics

Unfocused but intense, the friar's gaze is directed toward a distant abstraction and creates an evident contrast with the nobleman's focused stare, which is clearly fixed on the beholder. Meeting our eyes, the nobleman's stare demands that spectators pause and carefully consider Pacioli's activities and the contents of the painting. The friar is now absorbed in thought, but it is clear that he was just reading the open book and drawing on the slate with the chalk. In opposition to the stillness of his face and body, his hands are busy pointing; they press the viewers to guess the object of his thoughts. Conceived for the beholder, these direct pointers confirm that this panel is interactive, a window into a space, a cultural and intellectual space to be explored.

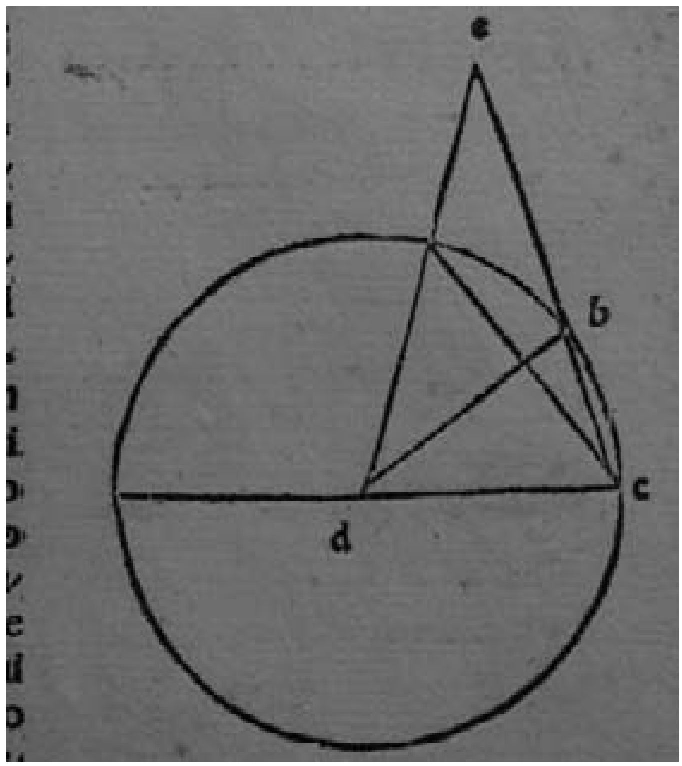
The process of sifting through the details of the evidence and following the seemingly abstruse leads that Pacioli and the painter provide makes these appear as "clues" in a complex system of signs purposely created to engage and aid the viewer to understand the painting's contents and meaning. As a road map to the intellectual dimensions of the panel, this system of signs is organized through the picture's visual structure (Fig. 1). The friar dominates the scene: his commanding triangular presence emerges from and visually rests on the table. The horizontality of the table is echoed by several visual sequences: the instruments in the foreground (eraser, square, pen, compass); the names (Euclid in the tablet and open book and implicitly, its printer, Erhard Ratdolt, Iaco.Bar. on the *cartellino,* Lucae Burgensis on the closed book) and the figures (tablet diagram, book diagrams, Platonic polyhedron) in the middle ground; and the sequence of faces organized in the upper level (the faces of the Archimedean polyhedron, of Pacioli, and of the nobleman). Understanding the conceptual associations that these sequences subtend requires much information about the numerous details included. Reading and deciphering them is a challenge, but the care invested by the painter in their depiction makes clear that they qualify the subject of this picture.

The identification of the sequence of names related to Euclid requires an explanation. By 1495 Euclid's *Elements* had been printed twice: the *editio princeps* by Ratdolt appeared in Venice in 1482; the second one, a virtual twin of the first edition, was issued in 1491 in Vicenza by Leonardus of Basel and Guglielmus of Pavia—both editions survive in more than one hundred and fifty exemplars. In addition to being a milestone of early printing, Ratdolt's edition is particularly

10 Dedicatory letter and first page of the text from Euclides, *Elementa geometriae,* Venice: Erhard Ratdolt, 1482, a1v–a2r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (text in the public domain; photograph by the author)



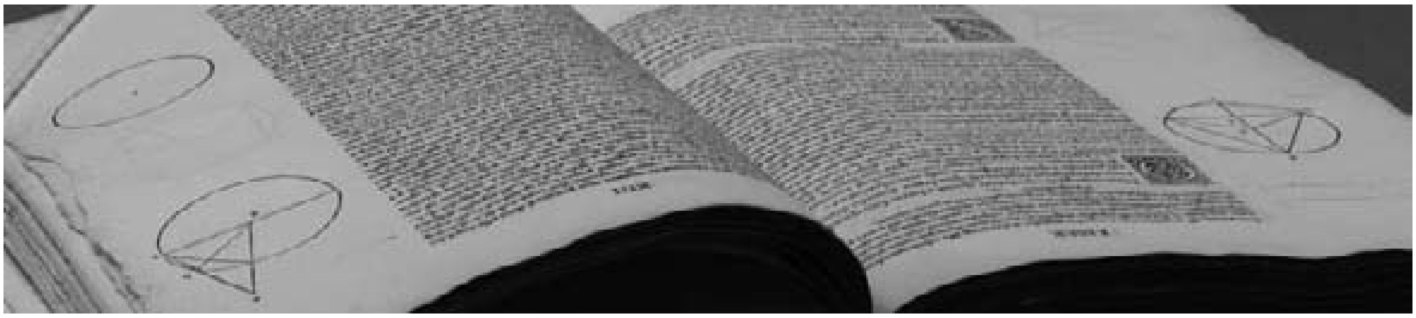
11 Detail of p3v from Euclides, *Elementa geometriae,* 1482. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (diagram in the public domain; photograph by the author). The actual length of the segment af is 1½ in. (3.8 cm).



12 Detail of p4r from Euclides, *Elementa geometriae,* Vicenza: Leonardus Achates de Basilea and Guilielmus de Papia, 1491. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (diagram in the public domain; photograph by the author). The actual length of the segment bd is 1 in. (2.5 cm).

important for the history of mathematics, and for the interpretation of our panel.[46] His almost five hundred metal imprints—about five times as many figures as most medieval manuscripts included—transformed the approach to the *Elements* and its contents: from a text that tested one's ability to follow intricate verbal statements, Euclid's magnum opus became the gymnasium for learning to argue in visual and graphic terms according to the grammar of geometry. To further emphasize the figures' importance, in the letter of dedication that serves as a foreword that Radtolt printed on the page facing the first of the text, he discussed their significance and claimed that he had both invented a new way of printing geometric figures and made the figures himself (Fig. 10). Despite the enormous investment that creating these figures required, he then sold the book at a relatively inexpensive price, transforming this volume into *mercanzia d'onore* ("honor-bringing goods") rather than merchandise good exclusively for monetary profit.[47] The following are Ratdolt's

13 Euclides, *Elementa geometriae,* 1482, p3v–p4r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (text in the public domain; photograph by the author)



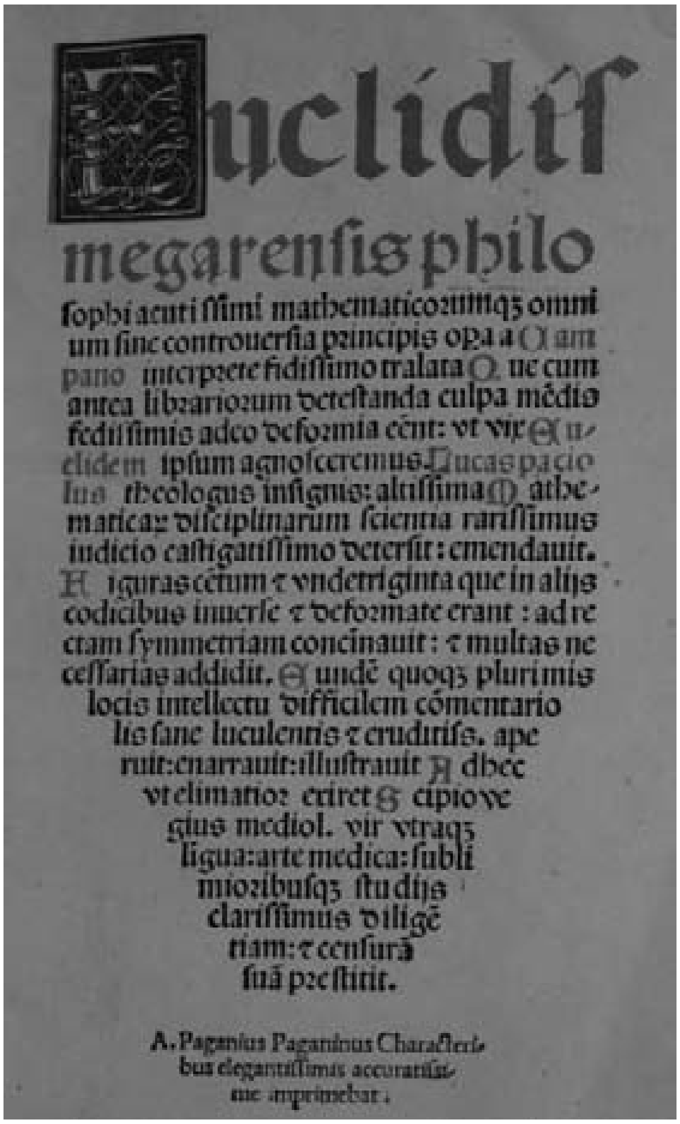
14 Euclides, *Elementa geometriae,* 1491, p3v–p4r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (photograph by the author)

own words from the dedicatory letter addressed to Doge Giovanni Mocenigo, where he explains the significance of his diagrams, which were in fact unprecedented for number and graphic quality:

> O most serene prince, in my own thoughts, I used to wonder why it is that in your powerful and famous city there are many works of ancient and modern authors being published, but none or few and of little importance are books of mathematics, even though this is a most noble discipline. After much thinking about this, I understood what the difficulty with mathematical texts is: until now, no one has found a way to make the geometric diagrams that abound in these works and without which nothing in mathematics can be properly understood. Therefore, because only this stands in the way of the advantages that these works offer to everybody, I applied myself and with great effort I made the figures, so that the geometric figures are printed with the same ease as the verbal parts of the *Elements*. And I hope that because of my contribution, soon the majority of books pertaining to the disciplines the Greeks call "mathemata" as well as the rest of the sciences will incorporate figures.[48]

The revolutionary claims of this statement and the impact of Ratdolt's geometric diagrams are relevant to the interpretation of our panel because they would surely have shaped the original discussion of the contents of the painting: on seeing what seems to be Ratdolt's famous book, viewers would have remembered its remarkable diagrams and the claims he made in the preface.[49] In turn, recalling his edition brought attention to the novelty of the diagrams in the painting, to the importance of their correctness, and to their significance for the developing mathematical sciences. Last but not least, the precise lines of the painted diagrams evoked the graphic precision of Ratdolt's metal impressions, which contemporary woodcuts, and specifically the cruder figures of the Vicentine edition, for those who knew both books, made all the more evident, praiseworthy, and memorable.

As observed, the foreground sequence of writing and drawing instruments is intended for the viewer. This is confirmed by the fact that the painter has not only placed these objects out of Pacioli's immediate reach but he has also given to the pen hanging from the table, to the compass, and to the square an orientation that renders these instruments for writing and drawing geometric figures directly available to the spectator. Yet: What are we to draw? and: Where are we to write? These questions have both a specific and a general answer, and it is important to begin by considering their specific solution. (I present it in summary form to avoid an extended mathematical exposition.) The specific answer to the question, "What and where are we to draw and write?" concerns the mistakes appearing in certain figures that accompanied Euclid's text in the first two editions, printed respectively in 1482 and 1491 (Figs. 11–14); two such problem figures are illustrated in the panel's diagrams. It should be noted that these mistakes call attention to the accuracy of geometric figures, a subject that was important to humanists preparing editions of ancient mathematical and scientific texts (many of which had lost their visual apparatus in their transcriptions by medieval scribes). This was also an issue very dear to Pacioli, who insisted on the correctness of his figures in the subtitle he included on the title page of his own edition of the *Elements* (1509): "Luca Pacioli, famous theologian with the most uncommon quality of having advanced knowledge in the mathematical sciences, on the most careful examination clarified and corrected one hundred and thirty figures



Euclidis
megarensis philo
sophi acutissimi mathematicorumq3 omni
um sine controuersia principis opa a C am
pano interprete fidissimo tralata Q ue cum
antea librariorum detestanda culpa mēdis
fedissimis adeo deformia essent: vt vix E u-
clidem ipsum agnosceremus. Lucas pacio
lus theologus insignis: altissima M athe-
maticar disciplinarum scientia rarissimus
iudicio castigatissimo detersit: emendauit.
F iguras cētum τ vndetriginta que in aliis
codicibus inuerse τ deformate erant: ad re
ctam symmetriam concinnauit: τ multas ne
cessarias addidit. E undē quoq3 plurimis
locis intellectu difficilem cōmentario
lis sane luculentis τ eruditiss. ape
ruit: enarrauit: illustrauit. A dhec
vt elimatior exiret S cipio ve
gius mediol. vir vtraq3
ligua: arte medica: subli
mioribusq3 studiis
clarissimus diligē
tiam: τ censurā
suā prestitit.

A. Paganius Paganinus Characteri-
bus elegantissimis accuratissi-
me imprimebat.

15 Title page from Euclides, *Elementa geometriae,* Venice: Paganinus de Paganinis, 1509, 1r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (text in the public domain; photograph by the author)

which in other books were reversed and imprecisely drawn, redrawing them to the correct composition while also adding many necessary ones" (Fig. 15).[50] The beholder is directed to this issue by the friar's main action: with the tip of his stick next to the line only halfway drawn, he seems to ask whether it is meaningful to extend the line to the circumference, implicitly questioning its function (Fig. 7).[51] Rather than extend it, the viewer should favor deleting this line with the eraser the artist has painted in the lower left. Placed as it is, this line is unnecessary in this figure and to the proposition in the text to which it belongs.[52]

More than simply underscoring the importance of the diagrams and the difficulty of correctly drawing them for advanced propositions of the *Elements,* Pacioli drew this line for two reasons. First, to invite beholders to look, to analyze the figure and the argument from a purely visual perspective, without invoking the infrastructure of references to the other propositions in the text, an infrastructure that is verbal and logical at its core. And, second, to refer to concrete examples of two other unnecessary lines appearing in the two printed editions of the *Elements* precisely at the pages painted here in the panel. One line connects the points DC in the figure for proposition 8 printed by Ratdolt (Fig. 11); the other one was mistakenly inserted in the 1491 edition (Fig. 12) in the diagram for the figure on the page facing the one the friar is considering (Fig. 4).[53] The complete explanation for this erasure involves geometric arguments; to follow them would distract us from addressing the more general question aimed at understanding the conditions under which a viewer of the later fifteenth century would have been able to reason about this line and understand that its presence is unnecessary for the demonstration.

To know or even recall the figures that Pacioli wants us to examine presupposes familiarity with advanced topics in Euclid's *Elements*; to make a case for the irrelevance and misplacement of the incomplete line implies mastery of the skills necessary to reason through geometric figures. Yet, although the panel makes proficiency in Euclidean geometry and visual reasoning desirable, it would be incorrect to assume that late quattrocento education familiarized every well-born person with advanced geometry, allowing one to detect the superfluousness of the line and argue against its presence. In fact, the standard fifteenth-century education covered the *Elements* only through book 5 (together with basic astronomical notions and some elementary celestial calculations); only someone who had pursued studies in advanced mathematics would have encountered the proposition and figures in question, which, as mentioned above, occur in book 13.[54]

However, in this regard, it is noteworthy that the Capodimonte panel celebrates the efforts of mathematical humanists, who, like Pacioli, were working to revive theoretical studies in mathematics while also propounding the nobility and usefulness of this discipline. Specifically, it sets forth their achievements by showcasing some of the main innovations and cutting-edge research topics in mathematics, while the objects depicted and issues raised propose a visual interpretation of the arguments for the nobility of this discipline and for the useful *forma mentis* (a cast of mind, set of mental habits) it provides. Polyhedra were the new research subject par excellence.[55] Also shown are the first ancient and the first modern mathematical books to be printed, respectively, Euclid's *Elements* and Pacioli's *Summa.* These are two key publications in the history of mathematics: the first canonized Campanus's version of Euclid's text and sanctioned the importance of the hundreds of figures that now accompanied it; the second epitomized—reformulating in modern terms—the achievements and techniques of the classical and medieval mathematical traditions. In addition, the diagrams carefully depicted in the *Elements* and on the slate instantiate the new modus operandi of mathematics, while underscoring that this discipline, as interpreted and resurrected by Renaissance humanists, is based on and fosters geometric and visual reasoning. Of course, as an expert on these subjects and as the editor and author of these books, Pacioli, fittingly, is part of this list of innovations. The painting, in addition to celebrating the person, turns him into an exemplum: he exemplifies the new, Renaissance mathematician, who is a court intellectual dedicated to theoretical endeavors, capable of connecting mathematics with philosophy and theology and of applying his knowledge to solve practical problems.[56] Finally, the fancy garments of the nobleman and the implied space of the scene identify the court as the natural environment of this new mathematics.[57]

We may ask whether and how an educated person in the late fifteenth century would have detected and argued about the interrupted line drawn by Pacioli on the slate tablet or against the two lines inappropriately included in the printed figures. The process requires one to draw and reason through diagrams and then write notes about them in the appropriate margins of one's copy of the *Elements,* and in doing so follow Pacioli in his thoughts. These questions connect to the other sets of visual associations concerning the books, diagrams, and names displayed by the panel. Together they represent the body of knowledge and skills necessary to consider the issues of these lines; they also implicitly suggest the pertinent educational ideals and an appropriate course of studies.[58] Immediately evident on the side of the slate tablet, the name of Euclid confirms to the beholders that the tome open on the table is a copy of the *Elements,* while its provocative presence on this tablet, rather than on the open book, suggests that in the interpretation of mathematical humanists, Euclidean geometry is not a form of knowledge to be recited or argued dialectically but one expressed through visual means; demonstrated graphically, it is diagrammatic and visual knowledge.[59] Moreover, the placement of the Euclid volume between the tablet and the *Summa* establishes their interrelation, even as it raises questions about their rapport, since the *Elements* is present both as a text and as a printed edition.

The appearance of the *Elements* as a printed book in the painting is important for the interpretation of the panel and therefore demands a brief discussion of the identification of the volume depicted by the painter. Most scholars have simply assumed that the artist copied a page from Ratdolt's edition; such an assumption, however, is incorrect (Fig. 13). Hannah Baader has recently argued that the larger blank space at the beginning of the second proposition and the absence of the letter A labeling the diameter in the upper left figure in both the 1491 diagram and the corresponding panel figure indicate that the painter chose to render the Vicentine edition of the *Elements* (Fig. 14).[60] Unfortunately, this is also incorrect. A careful examination of the painting reveals the absence of the two extra lines erroneously included in the 1491 diagrams discussed above.[61] This deletion together with the different proportions of the text and margins versus the size of the page are details that challenge her identification. But from this, it does not follow that the painter copied the pages of the 1482 edition, in spite of the fact that the remarkable finesse and exactness of the lines of the painted diagrams seem so clearly to match the graphic quality of Ratdolt's metal imprints. For one, the artist omitted two lines, changing Ratdolt's figure. What these apparently conflicting details tell us is that the painter did not intend to reproduce either edition.[62] The painter's first aim was to show Pacioli reading a *printed* text of the *Elements.* Secondly, with the slate tablet's diagram, which recalls the problems of the figures in the printed books, he prompted the expert viewer to discover that the friar is reading a corrected—albeit nonexistent—edition, since the panel's figures do not correspond to those printed in either 1482 or 1491.[63] Besides presenting corrected figures, the painter's depiction underscored graphic exactitude. Painting such remarkably fine lines required extraordinary care, and indicates that the artist and the person responsible for the picture's iconography understood the significance of graphic accuracy for geometric figures (and for reasoning through these figures)—and knew the significance of Ratdolt's achievement.

As depicted, the open book implies something else, too. In addition to demonstrating Pacioli's study of the volume, the marginalia visible on both pages—most of it highlighted in red ink—indicate the intended use of the pen offered to the viewer (Fig. 4). In other words, the marginalia serve as examples, encouraging beholders to go through their copy of the *Elements* to annotate its propositions and figures—especially those in the concluding sections, as the one seen in the panel, where the reader has to recall theorems and results from previous chapters. A closer look reveals verbal as well as graphic marginalia. While the verbal annotations suggest comments, references to other passages, or clarifications of the text itself, the various aligned segments likely visualize the proportions generated by the proof of the next proposition. These concern arcs, angles, and lines and illustrate Pacioli's ability to integrate different visual approaches to mathematical problems. Yet the main point proved by these marginalia is that the painted book is one that has been read line by line and understood completely. By visually asserting Pacioli's intimate knowledge of the *Elements*—the backbone of mathematics—the panel confirms the friar's authority as a mathematician entitled to teach the *Elements,* to comment on its contents, and to identify the direction that research in the field should take. Moreover, by picturing Pacioli as dominating the table with the *Elements* on it and busy working out the correct diagram for a complicated demonstration, the panel confirms the friar's intellectual position as heir to the classical tradition of mathematics and as leading exponent of the emerging movement of mathematical humanism. Pacioli's subsequent work, which included the publication of a revised edition of the *Elements* (1509) and the completion of an Italian translation (unfortunately lost), readily establishes the importance of his contribution to the tradition of the *Elements.* In fact, his publications and their fame ensured that the Capodimonte panel retained significant and specific intellectual value for those who owned it and who subscribed to its claims by displaying it.[64]

Pairing Two Books, Connecting Two Cultures

The simultaneous appearance of the *Summa* and the *Elements* in the painting is not casual, and the purposefulness of their relative placement is immediately evident to the attentive viewer. The prominent display of Euclid's text pushed the *Summa* to the sidelines; this displacement defies the conventions of portraiture by which an author is paired with his book. (Notably, it is this canonical combination that Baldi described.) Since in 1495 the *Summa* was Pacioli's only printed book and the best way to showcase his knowledge and contribution to the field of mathematics, the only reason to separate the friar from his tome and to pair him instead with Euclid's work was that in the picture's context Pacioli's public image would have benefited from this unconventional association. Indeed, the depiction of the printed *Elements* with revised diagrams qualifies Pacioli as a leading mathematical humanist. More importantly, the displacement of the *Summa* and its substitution by the *Elements* are unlikely inventions for a young painter; they seem choices that only Pacioli himself could have made, and they invite the audience to reexamine the panel to look for the evidence of his "hand" in the painting. Certainly, details like the half-drawn line in the slate's diagram and the corrected figures on the printed pages of the *Elements* suggest that the inventor of the picture's complex iconographic program was not "Jaco.Bar." but Pacioli himself.

This hypothesis finds support in the sort of signature the friar had the painter include on the panel, namely, the abbreviation of the *Summa*. The choice of the letters "LI R. LVC. BVR." to identify the work (Fig. 5), instead of an abbreviation that shortens the tome's printed title, was also an unlikely invention for the young painter and a solution that only Pacioli could prescribe. More importantly, the identification of the author and title of this book by spelling out the abbreviation on its side presses the viewer to think about the possibility of a double reference: one to the author of the book and one to the author of the painting's iconography. In studying the abbreviation in an effort to decode it and identify the book, one notices that the surface on which these letters seem impressed has a definite wooden solidity. That, in turn, invites a parallel with the material of the panel itself, making the letters of the abbreviation appear as Pacioli's signature to the painting, thus basically certifying that the friar was the mind behind its iconographic and intellectual program.[65] It should also be noted that the identification of the volume implies an important substitution that helps to define the cultural background of the panel's intended audience: because the letters "LI R. LVC. BVR." do not abbreviate the work's actual title, the tome becomes the *Summa* only in the eyes of those acquainted with the contents of the LI(ber) R(everendi) LVC(ae) BVR(gensis)—that is, with the contents of the book by the reverend Luca from the Borgo.[66] Among these were certainly persons aware of his toils and who possibly knew the friar himself, thus, a group that would have included members of many Italian courts, university students who attended his lectures, and those intellectuals to whom the *Summa* was addressed.[67]

Pondering this abbreviation leads to the scrutiny of the object itself, and this in turn makes the audience notice several important facts about the physical properties of both books on the table. The "solidity" of the pages of the *Summa* would have caught the eye of original viewers, making them realize that this book was brand new: only freshly cut paper can be so precisely compact. Also, anyone familiar with books would recognize the lavish binding as well as the straight edge of the long side of the volume: both are exceptional. Such a straight edge in the paper is peculiar: depending on the binding process, usually the resulting edge is either concave or convex; its straightness confirms the extra care in the binding and the fact that this book was probably never opened. The binding itself is princely: in addition to the elaborate center and corner bosses and the four clasps and catches, on the red leather covering the wooden boards is also visible an intricate decoration consisting of stamped motifs and linear patterns. Evidently, the conclusion sought by this depiction is that this is a brand-new book owned by an aristocratic collector and bibliophile.

There is more to learn from the inscription "LI R. LVC. BVR.": it would be inappropriate as a label if the book were owned either by Pacioli himself or by somebody who did not know him personally (or that his hometown is Borgo S. Sepolcro; today, Sansepolcro). By identifying the volume simply as *the* book of Luca from the Borgo, that is, Luca from town, it implies that its owner was acquainted with both the *liber* and *Luca Burgensis,* while the tome's physical characteristics indicate that he was a wealthy bibliophile. The physical characteristics of the open book suggest that the owner of the *Summa* owns also the Euclid. In addition to being bound with wooden boards (its smaller size required only two clasps, their catches visible on the left edge), this copy of Euclid's *Elements* has a firm flexible back with a multicolor silk headband woven with no fewer than four threads of different colors. Such lavishness confirms that this, too, cannot be Pacioli's own copy. Reasonably, both books belong to the same princely bibliophile who collected printed books, who received a copy of the *Summa* soon after it was published, and who had it bound sumptuously and labeled it simply as the book of Luca dal Borgo. This description fits precisely the character of Guidobaldo I, making it likely that the handsome, distinguished individual standing on the right is the duke of Urbino to whom Pacioli dedicated the *Summa* and to whom the friar taught mathematics.

The simultaneous presence and relative placement of the two books call attention to the relation between the two mathematical cultures they represent: the theoretical one of Euclid and the mixed—theoretical and applied—one of Pacioli. The fact that the *Elements* is open and the *Summa* closed suggests a priority of the first over the second. Conversely, the facts that the *Summa* is closer to the beholder and that it is placed on the same visual register as the eraser, square, pen, and compass, thus becoming another "tool" offered to the beholder, imply instead an order of study—certainly, Pacioli's volume would have served well as an introduction to Euclid's work.[68] In other words, the *Summa* is pictured here as a useful means to prepare students for the rigor of the *Elements,* even though the modern work also extends its predecessor by providing practical as well as theoretical applications of the classical material. These applications, too, would serve as effective stimuli for students to confront and master Euclid's text.[69]

Finally, the visual presentation of the two books implicitly qualifies the relation—in both content and form—between modern and ancient mathematics. In fact, the sort of intellectual filiation implied by the spatial proximity sanctions the research program of mathematical humanists as well as their chosen modus operandi, modeled after that of the ancients. Significantly, the main achievements and interests of the medieval mathematical tradition are absent from this panel.[70]

The theme of theory and practice brought forth by the multifaceted relation between the *Elements* and *Summa* is echoed by the opposition between the two polyhedra prominently included in the panel: the hanging Archimedean crystal and the wooden Platonic solid sitting atop the *Summa*

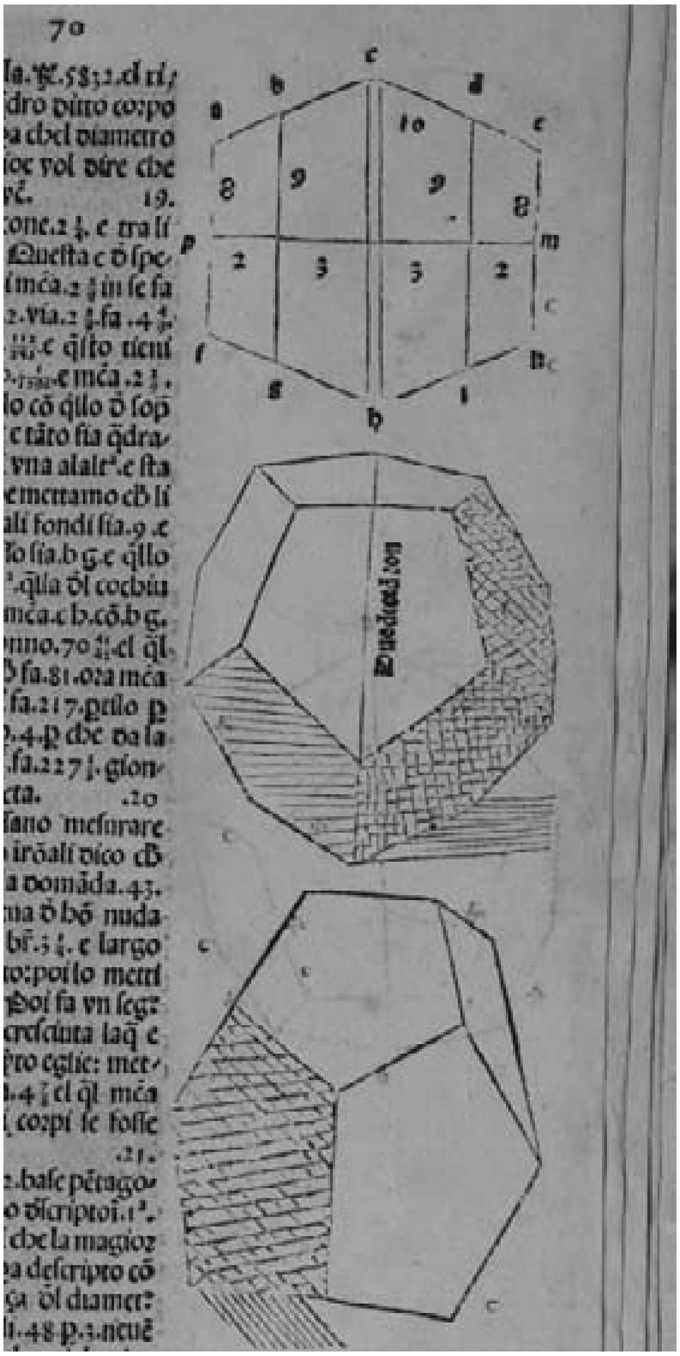
(Figs. 5, 9). The solidity of the wooden one, an object that the spectator could—figuratively speaking—easily pick up and handle, contrasts with the translucent, ideal composition of the glass one, which can only be contemplated. The rhombicuboctahedron seems to be out of our reach, as if existing in its own space—a space that appears truly incongruous to that of the painted scene because the two figures remain oblivious to its presence—and yet this crystal is also clearly situated geographically and defined culturally by its reflections, which suggest that it is hanging in Urbino's palace.[71] Moreover, the very depiction of this prominent and puzzling object attracts the eye and the mind of the beholder, who readily recognizes it as an impressive mathematical entity and a pictorial challenge.[72] As the spectator studies its faces and reflections, it becomes evident that painting this translucent polyhedron, even if drawn with the help of someone else's graphic model, ascribes to our painter not only advanced knowledge of perspective but also a remarkable capacity to think in three dimensions and visualize geometrically complex constructions. Certainly, what it measures and displays is not the ability to portray a real object but the skill to picture an abstract one—and because of the successful application of this skill, Jaco.Bar. is confirmed as a subscriber to the belief in the power and importance of geometric analysis proposed by Ratdolt and Pacioli. By partly filling the polyhedron with water, the artist demonstrated his ability to materialize, to realistically render in paint a complicated mathematical object. More important, with it he reifies in the eyes of the audience what was a paper concept probably far too complex in its three-dimensional structure to be realized as a hollowed crystal with a bottom hook placed on the inside and an opening made on the top so that it may be filled.[73] The pictorial challenge posed by its representation was admirably met by the painter, who matched Piero's skill and mathematical expertise, since this can be considered one of those "regular solids suspended from up high; from the lines, reflections, and shadows of these crystals one discovers how accomplished a painter Piero was."[74]

The comparison of the two polyhedra also brings to the fore an important issue concerning the panel's perspective system, a subject that naturally arises when studying this work. In examining the suspended crystal, the beholder discovers both the picture's vanishing point—where the orthogonals of its perspective system meet—and its preferred viewing point. In fact, the level of the water at the crystal's sides reveals that the vanishing point is located in Pacioli's head, at the height of the gentleman's eyes. Consequently, the ideal viewer looks down on the water that fills the crystal and on the friar himself. In short, we are on a par with the nobleman, whose eyes we meet straight on.

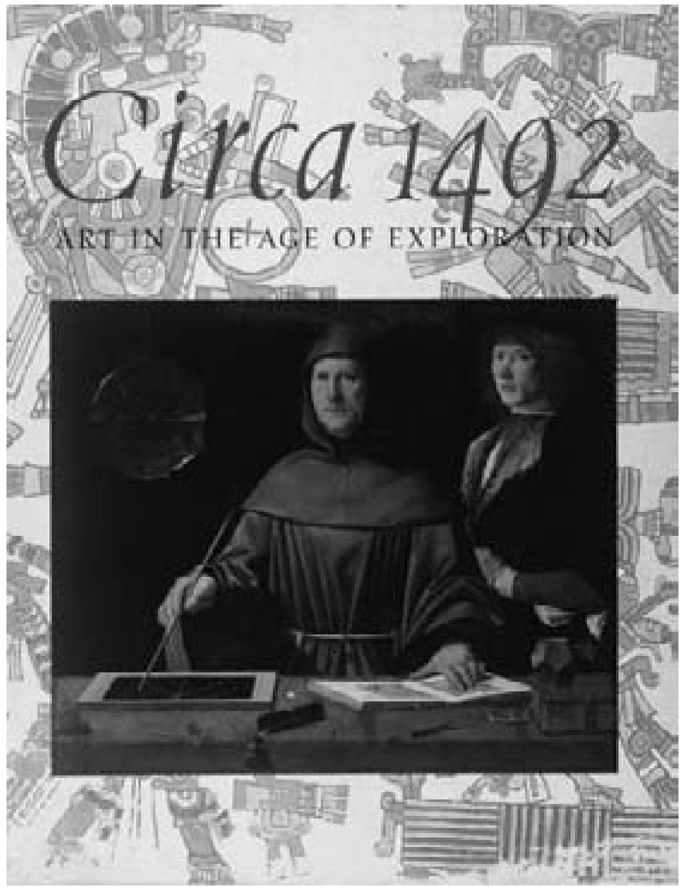
As the beholder peruses the objects on the table from this higher standing, a spontaneous doubt arises: Is the perspective of the dodecahedron correct? Specifically, if the painting's preferred viewpoint and vanishing point are higher than the water in the crystal, should not more of the top face of the wooden solid be visible (Fig. 5)? The answer to this instinctive question is another indicator of the level of mathematical knowledge and the kind of visual analysis skills demanded of the panel's target audience. As in the Platonic doctrine, here, too, appearances and shadows deceive: this perfect solid is drawn correctly. But to realize this implies that one has stereometric knowledge of Platonic solids, as well as the capacity to picture a dodecahedron in space and then to superimpose its skeletal structure onto the wooden body, to see that its top and bottom faces are not actually parallel.[75] Undoubtedly, few would have had the required geometric knowledge at their fingertips and be equipped with the analytic skills needed to solve this visual riddle and pass this test. Pacioli's *Summa,* which treats polyhedra and includes their figures—the first such illustrations to be disseminated by means of the printing press—would have prepared the audience by providing the necessary mathematical and visual information to "ace" this test—while *Divina proportione* would include skeletal structures of polyhedra (Figs. 16, 17).[76]

Having addressed the relevance of the writing and drawing instruments, diagrams, books, and faces, by way of summary it is useful to return to the visual sequences and reflect on the various associations proposed by the panel. Surely, after marveling at some of its remarkable pictorial passages and puzzling contents, the beholder embarks on the challenging but structured intellectual journey of understanding the painting, and along the way begins to comprehend the lessons the picture offers. Whether or not in the end the viewer correctly solves the various puzzles is less important than the realizations and connections made while trying. The audience discovers the practical and theoretical tools needed to follow Pacioli's thought and to meet the challenges posed by the panel's contents. In this process, the spectator is encouraged to draw diagrams for himself, to annotate geometric figures and propositions in his own books, and to sharpen his skills in thinking diagrammatically and analytically in two and three dimensions according to the grammar of geometry, learning this subject through the figures of the new printed editions. In doing all of this, the spectator becomes aware of being initiated and participating in the research program pursued by mathematical humanists, thinking and arguing in abstract visual terms, as did Euclid, Archimedes, Pappus of Alexandria, and other ancient mathematicians, engineers, and natural philosophers. Finally, the audience recognizes the implied educational goals aimed at creating the *forma mentis* advertised by the panel, realizing that they are supported by a modern course of advanced studies differing substantially from the traditional curriculum of the university. In academic lecture halls, Euclidean geometry and visual analysis and arguments remained peripheral subjects at best. Here, they are central.

A final word on the two figures and the artist. The complexity of the painting's contents displaces its focus from the friar. More than a portrait sitter, Pacioli stands as a symbol; his figure is employed to represent and explain the intellectual phenomenon of mathematical humanism and the critical skills of visual reasoning that this movement proposed. Considered in this way, the friar fits precisely the definition furnished by Bernardino Baldi, who aptly labeled Pacioli an illustrious *illustratore* (interpreter) of mathematical disciplines.[77] The real but nameless gentleman—likely Guidobaldo I, duke of Urbino—towering on the right also participates in the iconography of the painting: through his guise and stance, he sanctions its subject as courtly and noble, while also assuring its active reception with the



16 Detail of a page from the section "De corporibus regularibus" from Pacioli, *Summa de arithmetica,* 70r. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (text and diagram in the public domain; photograph by the author)

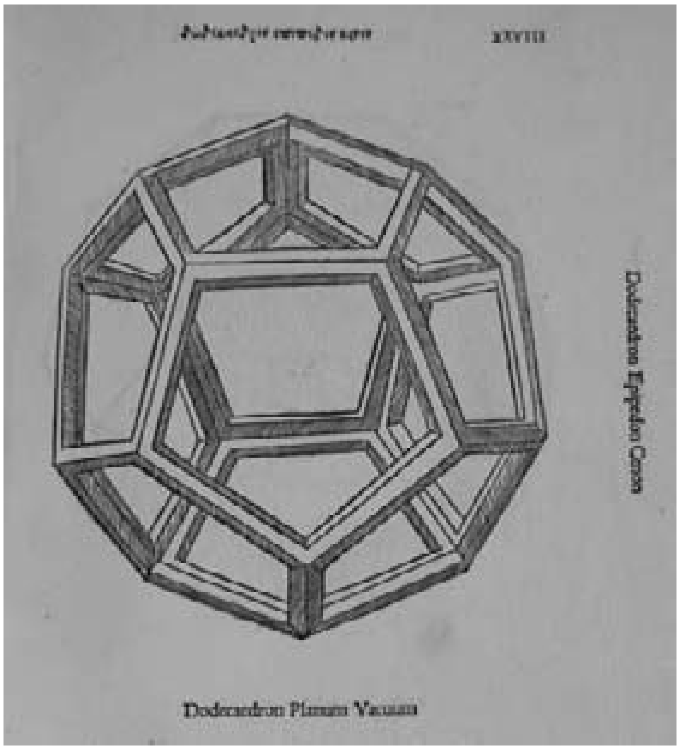


18 Dust cover of Jay A. Levenson, ed., *Circa 1492: Art in the Age of Exploration,* Washington, D.C.: National Gallery of Art, 1991 (cover art © National Gallery of Art; photograph by the author)

audience through his commanding stare. Finally, because of the precise and knowledgeable representation of the picture's contents, Jaco.Bar. appears not only as a promising artist who is already a *pictor doctus* (learned painter) but also as one who demonstrates that his art follows closely Alberti's recommendations that painting should integrate the visual grammar of geometry and that painters should be proficient in advanced mathematics.

The Panel in Perspective

The dust jacket of the catalog for the exhibition *Circa 1492: Art in the Age of Exploration* displays the Capodimonte panel on its front cover, setting it against a background of colorful motifs from a coeval Aztec religious manuscript (Fig. 18).[78] The editors' choice of making our painting into an epitome of European visual culture of the later fifteenth century and an icon of the Renaissance reflects not the presence of Pacioli—who remains an unsung hero of his culture—but the panel's remarkable display of perspective bravuras and its presentation of mathematics as a subject. This latter provides a fortunate connection to the traditional interpretation of the Scientific Revolution. Specifically, the seventeenth-century mathematization of nature, summarized by the Galilean dictum that "the Book of Nature is written in the language of mathematics, and its characters are triangles, circles, and squares," seems to find an early celebration of its language and sources in what is rendered in paint by our artist.[79] However, it is important to remind ourselves that it is only a posteriori that we can appreciate this connection, and that the painting did not aim to represent the integration of mathematics into Renaissance natural philosophy. Conversely, we should not lose sight of the novelty of the message imparted by the Capodimonte panel, nor of the fact that this picture was conceived as a culture maker. When it was painted, the panel celebrated the latest achievements of mathematical humanists; more importantly, it proposed their educational ideals and confidence in the power of geometric analysis as an essential intellectual value in education and as a desirable skill. Certainly, the knowledge of mathematics



17 Plate XXVIII from Pacioli, *Divina proportione,* Venice: Paganinus de Paganinis, 1509. Library of Congress, Washington, D.C., Rare Book and Special Collection Division (diagram in the public domain; photograph by the author)

and the proficiency in visual analysis that this painting advertises were still novelties in 1495. At that time, only a restricted group of humanists was familiar with them, but in the subsequent century, thanks to the power of the printing press and in no small part to Pacioli's *Summa* and Ratdolt's edition of the *Elements,* they would become crucial tools for natural philosophers, finding champions also among *mechanici,* that is, technicians and engineers.[80] The way in which the panel celebrates these intellectual innovations clarifies that they were not part of a new esoteric language, and that Euclidean diagrams and geometric solids were not intended to display the higher learning and skills acquired through an elitist education accessible only to a select few. On the contrary, they represent a class of analytic tools, truly the means for reasoning in visual and graphic terms available to all in their printed version. Moreover, the painting's open invitation to its spectators to interact with its contents indicates clearly that a proficiency in this new mode of reasoning is desirable for all Renaissance intellectuals; the books on the table and the tools necessary to learn from them are within reach of anyone.

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Notes

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1. For the basic bibliography on the painting, see Pierluigi Leone de Castris, ed., *Museo e Gallerie Nazionali Capodimonte: Dipinti dal XIII al XVI secolo; Le collezioni borboniche e post-unitarie* (Naples: Electa, 1999), 62–64. For a summary of the art historians' differing views about the painting, see de Castris's entry in *Rinascimento da Brunelleschi a Michelangelo: La rappresentazione dell'architettura,* ed. Henry Millon and Vittorio Magnano Lampugnani (Milan: Bompiani, 1994), 471–72. Most recently, the panel has been discussed by Enrico Gamba, "Pittura e storia della scienza," in *La ragione e il metodo* (Milan: Electa, 1999), 43–53; Hannah Baader, "Das fünfte Element oder Malerei als achte Kunst das Porträt des Mathematikers Fra Luca Pacioli," in *Der stumme Diskurs der Bilder: Reflexionsformen des Ästhetischen in der Kunst der Frühen Neuzeit,* ed. Valeska von Rosen et al. (Berlin: Deutscher Kunstverlag, 2003), 177–203; and Simone Ferrari, *Jacopo de' Barbari: Un protagonista del Rinascimento tra Venezia e Dürer* (Milan: Mondadori, 2006).
2. The painting's asymmetrical composition and unusual presentation of the figures confirm that this is not a traditional double portrait. On quattrocento double portraits—a pictorial genre that remains inadequately studied—see Charles Rosenberg, "The Double Portrait of Federico and Guidobaldo da Montefeltro: Power, Wisdom and Dynasty," in *Federico da Montefeltro: Lo stato / Le arti / La cultura,* ed. Giorgio Cerboni Baiardi et al., 3 vols. (Rome: Bulzoni, 1986), vol. 3, *La cultura,* 213–22. On the iconography of this latter painting, see Marcello Simonetta's discussion in *Federico da Montefeltro and His Library* (Milan: Y.Press, 2007), 102–9.
3. The emphasis given to geometric diagrams and their contribution to the panel's subject provide a cultural and intellectual context that helps explain their prominent appearance in several other coeval paintings, including, most famously, Sandro Botticelli's *Saint Augustine in His Study,* in the church of Ognissanti in Florence, and Raphael's *School of Athens,* in the Vatican Stanza della Segnatura. On the Vatican fresco, see Christiane Joost-Gautier, *Raphael's Stanza della Segnatura: Meaning and Invention* (New York: Cambridge University Press, 2002). On Botticelli's fresco, see Martin Kemp, "The Taking and Use of Evidence; with a Botticellian Case Study," *Art Journal* 44 (1984): 207–15. Kemp considers the diagrams seen in the open book (above Augustine) to be a display of the saint's intellectual pursuits and familiarity with cosmological studies, and to represent the highest forms of human knowledge. Several late-fifteenth-century artworks present the connection between geometric analysis and classical scientific knowledge that is evident in Botticelli's fresco, including two contemporary woodcuts, the much debated *Antiquarie prospettiche romane* (Rome: Andreas Freitag and Johann Besicken, ca. 1496) and Bernardus de Granollachs, *Lunarium* (Florence: Piero Pacini, 1496). Another representative example is found in a miniature in a *Natural History* codex completed for Giovanni Pico della Mirandola in 1481 and preserved in the Biblioteca Nazionale Marciana, Venice (Cod. Marc. lat. IV, 245 [= 2976]): diagrams appear in the books that Pliny the Elder is reading, serving to characterize the scholar's scientific studies and mode of reasoning. An illustration of this detail is available in Craig Kallendorf and Lisa Pon, eds., *The Books of Venice / Il libro veneziano* (Venice: La Musa Talìa; New Castle, Del.: Oak Knoll Press, 2008), 98.
4. The first, a document of 1631, describes it as "Un ritratto di un frate di San Bernardino con un giovane appresso vestito di pelliccia all'antica segnato in basso. Divo Principi Guido in Tavola" (A portrait of a friar of St. Bernard next to a young man wearing a fur coat in an antique style, signed at the bottom, His Highness Prince Guido on panel), and "Un Frate, che si dice sia il ritratto di Fra Luca dal Borgo, che non si sa di chi sia mano in tavola, che insegna Euclide al Duca Guido, della Guardaroba d'Urbino" (A friar, who is said to be the portrait of Fra Luca dal Borgo, by the hand of an unknown artist, teaching Euclid to Duke Guido from the *Guardaroba* of Urbino) (Archivio di Stato, Florence, Carte di Urbino Cl. II, Div. A, fasc. III). A 1654 inventory written by Bastiano Venturi, the duchess's secretary, notes, "Un quadro in tavola; un Frate di San Francesco con altra figura del Ghirlandaio" (A panel painting: a Franciscan friar with another figure by Ghirlandaio), and "Un quadro in tavola alto braccio 1 2/3, largo 2, di un Frate che insegna matematica. Del Ghirlandaio o di Luca Signorelli" (A painting on panel 1 and 2/3 arms high and 2 arms wide, of a Friar who teaches mathematics. By Ghirlandaio or Luca Signorelli). For the bibliographic references, see de Castris, *Museo e Gallerie Nazionali Capodimonte,* 62.

Despite their discrepancies, likely due to carelessness, these descriptions are in general agreement and support the identification of the second figure with Duke Guidobaldo. Their uncertainty about the panel's authorship raises questions about the *cartellino.* Reflectographic examinations have ascertained that the *cartellino* is original to the painting, but the lead content of its white makes it impossible to establish the authenticity of its inscription. See Martin Kemp's discussion of the painting in *Circa 1492: Art in the Age of Exploration,* ed. Jay A. Levenson (Washington, D.C.: National Gallery of Art, 1991), 244–46; and Marisa Dalai Emiliani, "Figure rinascimentali dei poliedri platonici: Qualche problema di storia e di autografia," in *Fra Rinascimento, manierismo e realtà,* ed. Pietro Marani (Florence: Giunti Barbera, 1984), 7–16, esp. 8–9.

5. Bernardino Baldi, *Le vite de' matematici: Edizione annotata e commentata della parte medievale e rinascimentale,* ed. Elio Nenci (Milan: FrancoAngeli, 1998), 330–45, at 344: "Pietro de Franceschi suo [Pacioli's] compatriota, pittore eccellentissimo e perspettivo, di mano di cui si conserva ne la guardarobba de' nostri serenissimi Principi in Urbino il ritratto al naturale d'esso frate Luca col suo libro avanti de la Somma Aritmetica et alcuni corpi regolari finti di cristallo appesi in alto, ne' quali, e da le linee, e da' lumi, e da le ombre, si scopre quanto Pietro fosse intendente ne la sua professione." A significant discrepancy with the Capodimonte panel is Baldi's mention of *several* hanging crystal solids. In this regard, it should be noted that crystal solids served as iconographic attributes to describe scholars dedicated to the study of ancient and modern scientific texts, as instantiated by three such solids in a sketch (Pushkin Museum, Moscow) by Vittore Carpaccio for the *Vision of Saint Augustine* (Scuola degli Schiavoni, Venice), where Saint Augustine is conflated with Cardinal Bessarion. This drawing is reproduced in Pietro Zampetti, ed., *Vittore Carpaccio* (Venice: Alfieri, 1963), 294, pl. 10 recto/verso.

6. Notably, Baldi disregards the wooden polyhedron on the lower right, despite the importance of polyhedra in Pacioli's connection to his patron in Urbino. Addressing Guidobaldo, the friar wrote in his *Summa de arithmetica geometria proportioni et proportionalita* (Venice: Paganinus de Paganinis, 1494), pt. 2, 68v: "[wooden polyhedra] are those forms the actual realizations of which Your Highness held in your hands in the remarkable palace of the Reverendissimus Cardinal, our patron, Monsignor of St. Peter in Chains, when Pope Innocent VIII came to visit in April of 1489, that is five years ago. Together with those, many other regular solids derived from these were crafted for Messer Piero de Valetarii of Genoa, bishop of Carpentras."

7. The hypothesis that "Iaco.Bar." worked from an original by Piero was first proposed by Adolfo Venturi, "Il più antico quadro di Iacopo de' Barbari," *L'Arte* 6 (1903): 95–96, and was supported by Fritz Heinemann in *Giovanni Bellini e i Belliniani* (Venice: Neri Pozza, 1962), 275. Maria Grazia Ciardi Duprè Dal Poggetto's influential counterargument rests on her skepticism that Pacioli would look as old as he appears in the panel when Piero was still alive; the painter died in 1492 when the friar was in his late forties. See Ciardi Duprè Dal Poggetto, "Il ritratto di Luca Pacioli e di Guidobaldo da Montefeltro," in *Piero e Urbino, Piero e le Corti rinascimentali,* ed. Paolo Dal Poggetto (Venice: Marsilio, 1992), 197–201, esp. 198. For her stylistic arguments supporting the attribution to Jacopo de' Barbari, see idem, "Jacopo de' Barbari e le Marche," in *Urbino e le Marche prima e dopo Raffaello* (Florence: Salina, 1983), 178–83. To my mind, the friar's age does not appear to invalidate the hypothesis of an original portrait by Piero, also because the painter of the Capodimonte panel would have been capable of making slight alterations.

8. Notably, the Museo e Gallerie Nazionali di Capodimonte continues to identify the painting as by "Jaco. Bar." In the hypothesized portrait, Piero could have depicted only the manuscript of the *Summa,* as it was published (1494) after the painter's death (1492). In our panel, the flatness of the *Summa*'s front side—not the result of overpainting—makes it look like a bookcase, but in 1495, beholders surely interpreted it as the published book.

9. Other pentimenti include the junction of the pages of the open book, the unnatural skin folds in the friar's hands and positions of the fingers, his upper lip, hood, and left eye. Despite the many afterthoughts, the naturalistic details reveal this painting—which is in good condition save for the blemish at the lower left—as the work of a talented artist, whose self-confidence is confirmed by the bold pictorial choice of placing the two figures against a black background while shining on them a crisp light. Before the cleaning done in the early 1990s (referenced in the painting by the square just below the middle knot in the friar's rope belt), the 1956 restoration corrected overpaintings and areas of paint loss, including the far right section, the fur of the gentleman's coat, the area beneath the *cartellino,* and the LI of the inscription on the closed book, while also filling numerous wormholes. Noteworthily, the three ghosts in the suspended crystal are not afterthoughts or byproducts of restorations. These are the two shadows evident in the lower right square face (next to these, there is an indentation in the wood that has been retouched in a lighter tone) and the elongated horizontal presence of a greenish tone, which bends correctly at the adjacent face and represents the reflection of the green tablecloth. These reflections prove that despite the obliviousness of the friar and his companion to the suspended polyhedron, it actually exists in their space.

10. The problems with the book's perspective are both general and particular (Fig. 4): the divergence between the text blocks and the page margins is exaggerated; the "LIBER" inscription, which misses its I, is imprecisely aligned; the shaft of the L is not parallel to the column of the text; the roman numeral XIII identifying the book number on the facing page is misaligned. Although seemingly consistent at first sight, the painting includes several perspective imperfections.

11. On this text, see Argante Ciocci, *Luca Pacioli e la matematizzazione del sapere nel Rinascimento* (Bari: Cacucci, 2003). Typographic evidence confirms that Pacioli worked on the *Summa* until 1509. See Derek Ashdown Clarke, "The First Edition of Pacioli's *Summa de Arithmetica* (Venice: Paganinus de Paganinis, 1494)," *Gutenberg-Jahrbuch,* 1974, 90–93.

12. The light source emanates from the upper left corner of the viewer's space, and the nobleman's glove, which is behind Pacioli, should be obscured by the shadow of the friar's body. While this might be intended, there are other problems with the painting's shadows. For instance, the two strings from which the ink container hangs should produce two shadow lines on the horizontal section of the green tablecloth, but only one is present. Another minor mistake concerns the binding of the closed book, which should include a part of the fourth clasp.

13. Three passages in the second figure demonstrate the painter's uncertainty and process of revision. Direct inspection reveals shadows underneath this coat of paint, suggesting a change in the jawline. Moreover, supported by the nobleman's hair mass, his semicircular hat—hardly visible in most reproductions—gives him a disproportionately large head with an awkward shape that exaggerates the left side of the back of the head. Finally, his left wrist is incorrectly drawn and anatomically impossible, unless one posits a remarkable, disfiguring fracture.

14. Radiographic and reflectographic images have been executed in 1956 and more recently for the exhibition *Piero e Urbino*; see Maurizio Seracini's technical report and its illustrations in Dal Poggetto, *Piero e Urbino,* 466–68.

15. Several scholars believe that the presence of the fly painted next to the last digit on the *cartellino*'s inscription questions its authenticity, because of the negative connotations attributed to this insect. However, the challenge of depicting flies was a topos in Renaissance art critical writings, including those of Leon Battista Alberti. For instance, adapting Pliny's story about Zeuxis and Parrhasios, Filarete in his *Trattato* narrates the anecdote of Giotto's flies that fooled his teacher, suggesting that artists used the representation of this minute insect to display their technical skills in trompe l'oeil effects. Reasonably, by painting the fly our painter provided connoisseurs with the occasion to praise his skills along lines well established in the art critical literature. Notably, despite the realism of the *cartellino* and the fly, a closer look reveals serious problems as well as the painter's inexperience. This *cartellino* cannot be folded according to the visible creases: folding it first longitudinally along the lower and shorter section and then vertically produces the folds as depicted, but then the curvature of the four sections becomes impossible. To be consistent, the lower left side should be bent down and not upward like the rest, as it is instead depicted. Moreover, in addition to the *cartellino*'s precarious resting position, a third problem concerns the uneven spacing of the "B AR" letters of the painter's name: it looks as if the name was written on the *cartellino* after it was already folded and placed in the scene. See *Filarete's Treatise on Architecture,* trans. John R. Spencer, 2 vols. (New Haven: Yale University Press, 1965), vol. 1, 309; and the entries 24–31 forming the section "Giotto's Fly and the Observation of Nature," in *Deceptions and Illusions: Five Centuries of Trompe l'Oeil Painting,* ed. Sybille Ebert-Schifferer (Washington, D.C.: National Gallery of Art in association with Lund Humphries, 2002), 163–79. For references to other primary sources, images, and a detailed study of the subject, see André Chastel, *Musca depicta* (Milan: Franco Maria Ricci, 1984), passim and 18.

16. For the basic information about Jacopo de' Barbari and his oeuvre, see Ferrari, *Jacopo de' Barbari*; and Mark J. Zucker's introduction to "Early Italian Masters," in *The Illustrated Bartsch,* vol. 24 (New York: Abaris Books, 1999), Commentary, pt. 4, 1–12. On the six-block woodcut view of Venice, see Giandomenico Romanelli et al., eds., *A volo d'uccello: Jacopo de' Barbari e le rappresentazioni di città nell'Europa del Rinascimento* (Venice: Arsenale, 1999). Finally, though a subject awaiting further examination, Jacopo's still lifes (a famous example is *Still Life with Partridge and Iron Gloves* at the Alte Pinakothek in Munich) are discussed by Eugenio Battisti, "Meditando sull'inutile," in *La natura morta in Italia,* 2 vols. (Milan: Electa, 1989), vol. 1, 38 n. 37.

17. Even though de' Barbari's pre-1500 oeuvre is not well established,

scholars have been confident about his Venetian training. Most recently, Ferrari has discussed the Venetian traits of the Capodimonte's painting in his entry for the catalog *Marco Palmezzano: Il Rinascimento nelle Romagne* (Milan: Silvana Editoriale, 2005), 178. John Steer has argued for Jacopo's presence in Vivarini's studio in the later 1490s in *Alvise Vivarini: His Art and Influence* (New York: Cambridge University Press, 1982), 92–96. Moreover, Creighton Gilbert made a case for the young artist being a considerable influence on the style of the older Venetian master; in addition to his article in the *Dizionario biografico degli italiani* (Rome: Treccani, 1964), 44–46, see Gilbert, "Alvise e Compagni," in *Scritti di storia dell'arte in onore di Lionello Venturi,* 2 vols. (Rome: De Luca, 1956), vol. 1, 277–308. For a review of the matter, see Sergio Guarino, "La formazione veneziana di Jacopo de' Barbari," in *Giorgione e la cultura veneta tra '400 e '500: Mito, allegoria, analisi iconologica* (Rome: De Luca, 1981), 186–98. Notwithstanding the fact that Piero had drawn similar polyhedra and that Pacioli himself had made models of them before 1498, Guarino favors a dating of 1498–1508 for our panel, maintaining that its solids depend on Leonardo's drawings from 1496–98.

18. See Creighton Gilbert, "When Did a Man in the Renaissance Grow Old?" *Studies in the Renaissance* 14 (1967): 7–32, esp. 28–30. "Vigennis" is unlikely to qualify the age of Guidobaldo, duke of Urbino, who was born on January 24, 1472, and was twenty-three years old in 1495.

19. No document connects Luca Pacioli to Jacopo de' Barbari, but it is possible that they met in Venice in 1494 when the friar supervised the setting and the corrections of the *Summa*. Jacopo's absence from the long list of Venetian perspective experts in the *Summa*'s introduction serves as circumstantial evidence that they did not meet before. For a history of perspective in later-fifteenth-century Venice, see Margaret Daly Davis, "Carpaccio and the Perspective of Regular Bodies," in *La prospettiva rinascimentale: Codificazioni e trasgressioni,* ed. Marisa Dalai Emiliani (Florence: Centro Di, 1980), 183–200.

20. Viewers familiar with the *Summa* would have recalled this initial, which was unconventional for its time: it is not decorative, nor does it present the stereotypical image of the mathematician or of the savant author—traditionally, a *doctus* (learned person) with a book in his hand or a teacher lecturing an audience. By presenting a *ritratto al naturale* (painted in the likeness), this woodcut avoided boxing Pacioli into stereotypes, fashioning him instead as a modern author and mathematical humanist. Moreover, this "iniziale parlante" (illuminated initial containing a portrait or a human figure) announced a text written by an intellectual who has command of the mathematical tradition, as well as an exposition of the material that integrates theory and practice as well as verbal and visual reasoning. Beholders would have appreciated the iconographic similarities between the woodcut and the painting. Inside the initial, as in the panel, Pacioli is shown set in a *studiolo*-like space, deep in thought rather than simply reading the book and prepared to express his thoughts through geometric forms. The awkward insertion of the two windows suggests that the woodcutter followed precise instructions, which demanded also that the open book's text blocks be matched with geometric figures (here, exemplified by a triangle and a circle), while leaving blank half of the right page for readers to write, or, as is suggested by the large compass, to draw on. More important, Pacioli is not passively absorbing knowledge from the established tradition but is instead interacting with and reacting to the book's contents, developing his own ideas from the results already established, ideas to be written through the visual language of mathematics. On *iniziali parlanti,* see Franca Petrucci Nardelli, *La lettera e l'immagine: Le iniziali "parlanti" nella tipografia italiana (secc. XVI–XVIII)* (Florence: Olschki, 1991), esp. 15; on the subject's historiography, see Nardelli's footnotes in "L'immagine e la lettera: Le lettere 'parlanti' nella tipografia veneziana ed italiana," in *Documentary Culture: Florence and Rome from Grand-Duke Ferdinand I to Pope Alexander VII,* ed. Elizabeth Cropper et al. (Bologna: Nuova Alpha, 1992), 307–16. Finally, the woodcut and panel comparison challenge the significance of the position of Pacioli's hood posited by Baader, "Das fünfte Element," 182–83; on the Franciscan uniform, see Giancarlo Rocca, ed., *La sostanza dell'effimero: Gli abiti degli ordini religiosi in Occidente* (Rome: Edizioni Paoline, 2000), passim, but esp. 58, 324–28.

21. Justus of Ghent's painting *Euclid* (1473–76, Palazzo Ducale, Urbino) instantiates the coeval interpretation of the prototypical mathematician and savant author. Part of the series of famous men in Guidobaldo's (1472–1508) *studiolo, Euclid* was paired with Federico's (1422–1482) teacher, Vittorino da Feltre, who is labeled "Mathematicus et omnis humanitatis pater."

22. Although early modern descriptions of the painting identify the nobleman with Duke Guidobaldo (Federico's son, born 1472), the name of Dürer has also been proposed. See Nick Mackinnon, "The Portrait of Luca Pacioli," *Mathematical Gazette* 77 (1993): 130–219.

23. Pacioli's collaboration with several prominent artists, including Piero and Leonardo da Vinci, lends support to the hypothesis that the second figure is a self-portrait of Jacopo de' Barbari, who would have benefited from recording a close connection with the friar. The controversial nature of this long-standing hypothesis is epitomized by the recent catalog *Nel segno di Masaccio: L'invenzione della prospettiva,* ed. Filippo Camerota (Florence: Giunti, 2001): the title given the painting there suggests parenthetically that the *allievo* (disciple) is Guidobaldo da Montefeltro, while in the entry for the painting, Margaret Daly Davis maintains that the Venetian attire of the second figure is better suited to Jacopo de' Barbari (137, pl. V). See also Venturi, "Il più antico quadro," 95; and Eugenio Battisti, *Piero della Francesca,* ed. Marisa Dalai Emiliani, 2 vols. (Milan: Electa, 1993), vol. 2, 565.

24. See the dedication of the *Summa* (Fig. 6). Duke Guidobaldo was also chosen as dedicatee by Piero della Francesca, Giorgio Valla, and Bartolomeo Zamberti for their respective mathematical works; in 1499, Aldus Manutius dedicated to him his edition of classical astronomical works, *Astronomici veteres graeci et latini,* and the *Hypnerotomachia Poliphili.*

25. The painting's celebration of mathematical studies and printed books is significant in the Urbinate context. Notably, the absence of manuscripts points to the different approach to book collecting of Federico and Guidobaldo. Unlike his father, for whom, as Baldassare Castiglione recalled in *Il libro del cortigiano,* manuscripts were the "most important treasures in his great palace," Guidobaldo actively collected printed books. The painting's display of printed mathematical books might reflect his education—tutored by Pacioli since 1482, Guidobaldo likely learned geometry on the first edition of Euclid—as well as the legacy of Urbino as temporary repository for Cardinal Bessarion's library and host to the Academia Bessarionea; the cardinal and his protégé Regiomontanus believed that printing classical texts was key for the recovery of the ancients' scientific knowledge. See Castiglione, *Opere,* ed. Carlo Cordiè (Milan: Ricciardi, 1960), 17; and Luigi Michelini Tocci, "La formazione della biblioteca di Federico da Montefeltro: Codici contemporanei e libri a stampa," in Baiardi et al., *Federico da Montefeltro,* vol. 3, *La cultura,* 9–18.

26. See Gamba, "Pittura e storia della scienza," 48–50.

27. Ibid., 47.

28. In perfect solids, the same regular polygon forms all the faces (the cube is the simplest example of a perfect solid). In semiperfect solids, different regular polygons form the faces. The five perfect or Platonic solids can be inscribed in and circumscribed by a sphere. The thirteen semiperfect or Archimedean solids can only be circumscribed. In our panel, the wooden solid is a dodecahedron; regular pentagons form the faces of this Platonic solid. The suspended crystal is a rhombicuboctahedron; eight equilateral triangles and eighteen squares form the faces of this Archimedean solid. Notably, its attachment by wire to the bottom face wire-attachment differs from that seen in either the manuscript or printed versions of Pacioli's *Divina proportione*. In the 1509 edition (Venice: Paganinus de Paganinis), where the rhombicuboctahedron appears as plates xxxv and xxxvi, the solids float in space; in the three manuscripts, the polyhedra hang from their "cartellino" by means of a string attached to their top face or angle. See Vittorio Sgarbi, "De Divina Proportione," *FMR* 9 (1982): 103–15; and for a facsimile of the Ambrosiana manuscript, see Luca Pacioli, *Divina proportione* (Milan: Silvana Editoriale, 1982).

29. Reflectographies show the silhouette of a stick figure in the crystal's front triangular face; see Dal Poggetto, *Piero e Urbino,* 467–68. Conversely, the X-ray published by Guarino ("La formazione veneziana di Jacopo de' Barbari," 196) shows no figure.

30. Baader, "Das fünfte Element," 178–79, is the only scholar who has noticed that the pen is offered to the beholder.

31. On mathematical humanism, a subject awaiting further study, see Paul Lawrence Rose, *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo* (Geneva: Librairie Droz, 1975). Art historians are familiar with "mathematical humanism" from André Chastel's authoritative survey *I centri del Rinascimento: Arte italiana 1460–1500* (Milan: Feltrinelli, 1965), 41ff. He identifies three strands of humanism and specifies that the mathematical one "finds its most important base in Urbino" (41), noting that "the case of Luca Pacioli is not isolated: on the contrary, it well represents the intellectual environment of the quattrocento, an environment in which theory and practice walk hand in hand without, however, adapting themselves to one another perfectly" (47, 49). In Chastel's opinion, the Capodimonte painting represents "the perfect image of the mathematical humanism flourishing in Urbino and of its status there," but his qualification of the "green carpet" as "of a magician of science" transforms Pacioli into a learned illusionist.

32. Giorgio Vasari, *The Lives of the Most Excellent Painters, Sculptors and Architects,* trans. Gaston du C. de Vere, ed. Philip Jacks (New York: Modern Library, 2006), 159; and idem, *Le vite de' più eccellenti pittori, scultori ed architetti,* ed. Gaetano Milanesi, 8 vols. (Florence: Sansoni, 1878), vol. 2, 487. Pacioli in fact acknowledged in print his debt to Piero della Francesca; see Ciocci, *Luca Pacioli,* 94–109, 141–48.

33. The bias against Pacioli's intellectual contributions exemplifies the Whiggish approach to the history of science, which is epitomized by Gino Loria's presentation of the study by Girolamo Mancini, "L'opera *De corporibus regularibus* di Pietro Franceschini detto Della Francesca usurpata da Fra Luca Pacioli," [*Memorie della Regia Accademia dei Lincei, classe di scienze morali, storiche e filologiche* 312 (1915): 441–580]: "The aim . . . of the history of science is to find the genesis [of ideas], clarifying with complete impartiality the merits and faults of intellectuals; therefore, the undertaking pursued by Mr. G. Mancini of restoring that which was injustly taken from the famous painter of Borgo San Sepolcro [that is, Piero della Francesca] should be applauded and supported" (445).

34. Ciocci, *Luca Pacioli*. Still useful for a biographical account of the friar's life is Emmett Taylor's *No Royal Road: Luca Pacioli and His Times* (Chapel Hill: University of North Carolina Press, 1942).

35. In the later fifteenth century several remarkable intellectuals resided in Venice. Noteworthy among these, Giorgio Valla and Ermolao Barbaro played a leading role in the humanist efforts to recover the visual language of ancient science. To mathematically inclined humanists, Venice offered unique resources, ranging from an intellectually stimulating environment and academic freedom, fostered also by the Scuola di Rialto and that of S. Marco—two institutions of higher education that operated in complete independence of the Paduan *studium* and traditional university curricula—to the availability of unique primary sources in the libraries of Cardinal Bessarion, of the monastic houses of S. Giorgio and of SS. Giovanni e Paolo, and of private bibliophiles. On these sources, their use, and their import, see Marino Zorzi, *La Libreria di San Marco: Libri, lettori, società nella Venezia dei Dogi* (Milan: Mondadori, 1987), chaps. 3–5; and idem, "Dal manoscritto al libro," in *Storia di Venezia,* 12 vols. (Rome: Istituto dell'Enciclopedia Italiana, 1996), vol. 4, 817–954. For an overview of later-fifteenth-century Venetian humanism, see Vittore Branca, "L'umanesimo veneziano alla fine del quattrocento," in *Storia della cultura veneta,* vol. 3, pt. 1, *Dal primo quattrocento al Concilio di Trento,* ed. Girolamo Arnaldi and Manlio Pastore Stocchi (Vicenza: Neri Pozza, 1980), 123–75.

36. See Vasari, *Le vite,* vol. 2, 488. On the cultural context of Pacioli's hometown, see James Banker, *The Culture of San Sepolcro during the Youth of Piero della Francesca* (Ann Arbor: University of Michigan Press, 2003).

37. On fifteenth-century Venetian intellectuals, see Margaret King, *Venetian Humanism in an Age of Patrician Dominance* (Princeton: Princeton University Press, 1986). Their appreciation of mathematical and perspective studies is demonstrated by the commission of the mosaics composed and laid by Paolo Uccello (1425–30) on the floor of the basilica of S. Marco, which include a "duodedron elevatus," as Pacioli called it, that is, the fourth Platonic solid modified so that from each of its twelve pentagonal faces extends a five-sided pyramid.

38. This biographical information depends on a long passage in the *Summa* (67v) where Pacioli describes his career's residencies, movements, and writings. For the Scuola di Rialto, see Bruno Nardi, "La Scuola di Rialto e l'umanesimo veneziano," in *Umanesimo europeo e umanesimo veneziano* (Florence: Sansoni, 1963), 93–139; and Fernando Lepori, "La Scuola di Rialto dalla fondazione alla metà del cinquecento," in *Storia della cultura veneta,* vol. 3, pt. 2, 539–605.

39. See Pacioli, *Divina proportione* (1509), 29v, which includes a passage that establishes his competence in architecture—an art that was considered a mathematical discipline and one of the liberal arts—and his credentials as critic of Alberti.

40. Arnaldo Bruschi has aptly described Pacioli as "the typical university teacher of the humanism period." Bruschi, *Scritti rinascimentali di architettura* (Milan: Il Polifilo, 1978), 29.

41. At the end of book 4 of his edition of the *Elements* (1509), 31r, Pacioli listed the names of ninety-four of the approximately five hundred attendees. *Divina proportione* includes two other treatises: *Tractalo delarchitettura* and (Piero's) *Libellus.* Finished by 1498, it circulated in manuscript form—the copies dedicated to Ludovico Sforza, duke of Milan, Galeazzo Sanseverino, and Pietro Soderini still survive. For the text of the opening lecture of this course, see Nardi, "La Scuola di Rialto," 114–16. In addition to a special relationship with Paganino Paganini, in Venice Pacioli had many supporters, as suggested by the *Summa*'s acknowledgments.

42. Pacioli dedicated a substantial section of the *Summa* to techniques and problems of accounting and applied mathematics. On this material, see Basil Yamey, "Luca Pacioli, la *Summa* e il *De scripturis,*" in *Trattato di partita doppia,* by Luca Pacioli, ed. Annalisa Conterio, introduction and commentary by Yamey (Venice: Albrizzi, 1994), 11–34.

43. Although Pacioli completed the Latin edition and Italian translation afterward, very likely he was working on them already in 1495.

44. Several manuscripts of ancient mathematical and scientific texts that included a wealth of geometric diagrams (including Biblioteca Apostolica Vaticana Urb. lat. 1329, Vat. lat. 2224, and Urb. lat. 261) had been produced in Rome in the 1450s and 1460s. See Marco Buonacore, ed., *Vedere i classici: L'illustrazione libraria dei testi antichi dall'età romana al tardo medievo* (Rome: Fratelli Palombi, 1996), 381–83, 394–98, and 408–12; and Noel Swerdlow, "The Recovery of the Exact Sciences of Antiquity: Mathematics, Astronomy, Geography," in *Rome Reborn: The Vatican Library and Renaissance Culture,* ed. Anthony Grafton (New Haven: Yale University Press, 1993), 125–67.

45. Pacioli's understanding of the power of images and of the printing press to disseminate his ideas and mode of thinking was matched by his decision to discuss the most diverse mathematical subjects, ranging from the simplest accounting principles to abstruse problems in solid geometry, in the vernacular. Though composed in Italian, the *Summa* remained above the intellectual reach and probably outside the interests of merchants and shopkeepers, as it assumed that readers have received an advanced education and mastered more than the rudiments of Latin. Pacioli likely wrote it in the vernacular in an attempt to place the study of (the new) mathematics outside the realm of the university and the academic tradition, seeking and reaching an audience of court members, humanists, and practitioners of the applied arts.

46. The *editio princeps,* Euclides, *Elementa geometriae* (Venice: Erhard Ratdolt, 1482), appeared at a time of renewed interest in advancing geometric knowledge and in reestablishing the *Elements* as a cornerstone of education. See Gino Arrighi, "La fortuna di Euclide ovvero la geometria in Occidente durante il Medievo," *Atti e Memorie della Accademia Nazionale di Scienze, Lettere e Arti di Modena* 7, no. 6 (1988–89): 69–76. Arrighi summarized the medieval status of the *Elements* with these words (271): "as an overall historical judgment, we can affirm that the diffusion of the *Elements,* basically nonexistent before the effort of Abelard, afterward had a positive trend; however, for an entire century, that is, until the appearance of the works of Leonardo [Pisano, better known as Fibonacci] which offer practical applications, we do not see any new impulse during the two centuries separating the *Pratica geometrica* and the activities of Piero [della Francesca]." The three translations completed in the twelfth century assured the survival and availability of Euclid's text during the Middle Ages. Despite sustained manuscript traditions, scholastics remained uninterested in mastering advanced topics in geometry and extending the classical tradition. For the text's medieval history, see John Murdoch, "Euclid," in *Dictionary of Scientific Biography,* ed. Charles Gillispie, 16 vols. (New York: Scribner's, 1971), vol. 4, 437–59; and Menso Folkerts, "Aritmetica e geometria" and "La matematica nell'Europa latina," in *Storia della scienza,* vol. 4, *Medioevo, Rinascimento* (Rome: Istituto dell'Enciclopedia Italiana, 2001), 141–49, 313–23.

47. I am appropriating the expression "mercanzia d'onore" from Amedeo Quondam, " 'Mercanzia d'onore,' 'Mercanzia d'utile': Produzione libraria e lavoro intellettuale a Venezia nel cinquecento," in *Libri, editori e pubblico nell'Europa moderna,* ed. Armando Petrucci (Bari: Laterza, 1977), 51–104.

48. Erhard Ratdolt, dedication, in Euclides, *Elementa geometriae,* a1v. The original Latin reads as follows: "Solebam antea serenissime princeps mecum ipse cogitans admirari quid cause esse quod in hac tua prepotenti et fausta urbe cum varia auctorum veterum novorumque volumina quotidie imprementer in hac mathematica facultate vel reliquarum disciplinatum nobilissima aut nihil aut parva quedam et frivola impressorum copia qui in tua urbe agunt viderentur inpressa. Haec cum mecum sepius discuterem invenebam id difficultate operis accidisse. Non enim adhuc quo pacto schemata geometrica quibus mathematica volumina scatent ac sine quibus nihil in his disciplinis fere intellegi optime potest excogitaverant. Itaque cum hoc ipsum tantummodo communi omnium utilitati que ex his percipitur, obstaret mea industria non sine maximo labore effeci, ut qua facilitate litterarum elementa imprimuntur, ea etiam geometrice figure conficerentur. Quamobrem ut spero hoc nostro invento he discipline quas methemata greci appellant voluminum copia sicuti relique scientie brevi illustrabuntur."

49. On this book, its diagrams, the printer's introduction, and their intellectual context, see Renzo Baldasso, "La stampa dell'editio princeps degli *Elementi* di Euclide," in Kallendorf and Pon, *The Books of Venice,* 61–100.

50. Luca Pacioli, title page, in Euclides, *Elementa geometriae* (Venice: Paganinus de Paganinis, 1509), 1r: "Luca Paciolus, theologus insignis, altissima mathematicarum disciplinarum scientia rarissimus judico castigatissimo detersit et emendavit. Figuras centum et undetriginta quae in aliis codicibus inverse et deformate erant ad rectam symmetriam concinnavit et multas necessarias addidit."

51. Extended to the circumference, this line would approximate the side of an inscribed hexagon. The mark that follows the end of the line is generic and not a letter.

52. This proof can be easily carried out in visual terms. The claim is that the square of the side of the equilateral triangle inscribed in a circle is three times the square of its radius (Fig. 11). The proof proceeds in the following way. The diameter AE bisects the angle A, the side BC at

F, and the arc BC at E. The arc BE is one-sixth of the circumference, while the chord BE is the side of the inscribed hexagon; note that the chord BE is drawn in the printed editions but absent in the painting. Beholders familiar with geometry immediately know that this chord and side of the hexagon is equal to the radius. (Another visual step makes this evident: the triangle BDE has two radii as sides, and the angle that they form at D is of 60 degrees, being opposite to the side of a hexagon; therefore, the other two must also be 60-degree angles, and this clarifies that this triangle is equilateral, and that BE is equal to the radius.) Consider areas next. The Pythagorean theorem (from book 1) indicates that the square built from the diameter AE is equal to the sum of the squares built on the sides AB and BE. Moreover, the square of the diameter is four times the square of the radius. By substitution, the sum of the squares of the two sides of the triangle is equal to four times the square of the radius. Since the short side of the triangle is equal to the radius, by substitution and subtraction, the square of the side of the equilateral triangle is three times the square of the radius in which it is inscribed. Most importantly, the entire proof can be followed through a series of visual steps carried out mentally, without actually drawing a line or writing an equation. This is visual reasoning at its best.

53. In the figure with the equilateral triangle inscribed in a circle, the difference between the painted and the printed diagram amounts to the presence of three extra lines: two radii joining the lower angles of the triangle to the center of the circumference and the chord BE (Figs. 11, 12, 4). If the proposition is approached visually, the lines are unnecessary for the proof. The one joining DB leads to the realization that the triangle BDE is one of the six that compose the inscribed hexagon. However, the other line is simply confusing, because it invites readers to try to work out the theorem by comparing the areas of the triangles formed by the radius and applicable also to the square of the side of the equilateral triangle. This is a misleading approach to the theorem.

54. See Paul Grendler, *Schooling in Renaissance Italy: Literacy and Learning, 1300–1600* (Baltimore: Johns Hopkins University Press, 1989), esp. chap. 11. Other coeval artworks confirm the expectations of the panel, including the figure of Geometria on Antonio Pollaiuolo's funerary monument for Pope Sixtus IV (1493), which epitomizes the growth of interest in the mathematical disciplines in the second half of the fifteenth century. On the funerary monument (completed in 1498) of Pope Sixtus IV (d. 1484), Geometry is one of the subjects whose pursuit is considered virtuous; her nine companions include Arithmetic and Optics. See Alison Wright, *The Pollaiuolo Brothers: The Arts of Florence and Rome* (New Haven: Yale University Press, 2005), 359–87, esp. 381–82.

55. The coeval fascination with polyhedra is instantiated in intarsia (Gubbio and Urbino), mosaics (Paolo Uccello's mosaics in the basilica of S. Marco), drawings (Piero della Francesca's *De quinque corporibus regolaribus*), and prints (Dürer's *Melencolia I*). On this topic, see Enrico Gamba, "Piero inventore dei poliedri come 'genere,'" 477–78, and Vico Montebelli, "Piero, la matematica e i poliedri," 479–85, both in Dal Poggetto, *Piero e Urbino*. See also Judith V. Field, "Rediscovering the Archimedean Polyhedra: Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Albrecht Dürer, Daniele Barbaro, and Johannes Kepler," *Archive for History of Exact Sciences* 50 (1997): 241–89.

56. Through the two polyhedra and, obliquely, through the text following Proposition 8 that discusses the pyramid (the first Platonic solid), the panel may also illustrate Pacioli's interest in Platonism. Baader, "Das fünfte Element," addresses the potential connections, but see also Edoardo Mirri, "Elementi di filosofia platonica in Luca Pacioli," in *Filosofia e cultura in Umbria tra Medioevo e Rinascimento: Atti del IV convegno di studi umbri; Gubbio 22–26 maggio 1966,* ed. Francesco Ugolini (Gubbio: Università degli Studi di Perugia, 1967), 377–89.

57. Even if the second figure were not a portrait of Guidobaldo da Montefeltro, the reflections of the princely palace visible in the crystalline solid are oblique but unmistakable references to the Urbino court, which was the leading center in the development of mathematics. Both Federico and Guidobaldo da Montefeltro were interested in theoretical and applied mathematical studies. In addition to Rose, *The Italian Renaissance of Mathematics,* see Antonio Manno, "Architettura e arti meccaniche nel fregio del Palazzo Ducale di Urbino," in Baiardi et al., *Federico da Montefeltro,* vol. 2, 89–104.

58. Half a century earlier, in the pages of *De pictura* and *Ludi matematici,* Alberti—a friend and perhaps a mentor of Pacioli—argued for the integration of mathematics and visual reasoning in the humanistic curriculum. See Gino Arrighi, "Leon Battista Alberti e le scienze esatte," in *Convegno internazionale indetto nel V centenario di Leon Battista Alberti* (Rome: Accademia Nazionale dei Lincei, 1974), 155–212.

59. The labeling of the slate tablet and its chalk-drawn contents "Euclid" should not be interpreted as a tacit attack on the validity of Euclidean theorems. Similarly, the pen and the ink container do not represent a polemical approach to the mathematical knowledge of the ancients. On the contrary, the slate tablet and the pen, being also learning tools, imply that the Euclidean tradition is alive; these writing and drawing instruments give the opportunity, or better, they invite the beholder to extend this tradition.

60. The printed letters painted for the book's diagrams actually are not letters but general marks, while those of the figure drawn on the slate tablet are meant to be readable (and be read). For Baader's arguments, see "Das fünfte Element," 184–85. She cites Jay A. Levenson as support, but in his dissertation he had noted that the open book could be either edition, as he considered the 1482 and 1491 versions to be virtually identical volumes; see Levenson, "Jacopo de' Barbari and Northern Art in the Early Sixteenth Century" (PhD diss., New York University, 1978), 304. Although I disagree with Baader's conclusion, her comparison between the painting and the early editions of the *Elements* was a starting point for this study. I also disagree with Gamba's claim that "the correspondence between the painted book and the original one is intentionally perfect in all its details" ("Pittura e storia della scienza," 46).

61. The line in the second figure that extends from C to the circumference does not have any relevance for the proof of this proposition or of its reverse conclusion (Fig. 12). This line does not appear in subsequent editions of the *Elements,* including Pacioli's edition of 1509 and the 1504 one prepared by Bartolomeo Zamberti, who interpreted the original proposition differently, producing also a completely different figure. In my understanding, the line in question has no clear geometric function or value as a proportional term to be compared with the other elements of the figure.

62. In addition to the "imperfections" already mentioned, other details clarify that the painter did not pursue an exact portrayal of either edition. For instance, a notable discrepancy is the mistake in the "Liber" heading where LiB was fused into two letters, which is not the case in either edition (Figs. 4, 13, 14). Similarly, the painted book misrepresents both editions because book 13 and this specific passage are found toward the end of the volume, which in the painting is opened at a halfway point instead.

63. Notably, Pacioli's edition (1509) revised the second figure of the 1491 edition but copied the first, including the lines omitted in the panel.

64. According to Menso Folkerts, the friar worked on an Italian translation of the *Elements* between 1494 and 1497. See Folkerts, "Luca Pacioli and Euclid," in *Luca Pacioli e la matematica del Rinascimento,* ed. Enrico Giusti (Città di Castello: Petruzzi, 1998), 219–33. Pacioli himself advertised this translation as forthcoming in the *Divina proportione*'s dedicatory letter addressed to Pietro Soderini.

65. The double meaning of the Latin word *liber*—"book" as well as "bark" of a tree—facilitates this connection.

66. The significance of this inscription is underscored by the fact that instead of being upside down, as appropriate for a book in this position, it is written for viewers to read. This inscription abbreviates neither the title (*Summa de arithmetica geometria proportioni et proportionalita*) nor the text's opening sentence. Following Venturi, "Il più antico quadro," Guarino ("La formazione veneziana di Jacopo de' Barbari," 196) proposed an alternative reading, "Liber Regularum" instead of "Liber Reverendi," and identified the work as a codex of *Divina proportione.*

67. Several passages in the text shed light on the *Summa*'s target audience. For instance, in "Tractatus Geometrie," *Summa*'s concluding section, 1r, Pacioli writes, "as I anticipated at the beginning, the second part of the present work . . . will treat continuous quantities, i.e., the application of geometry to practical problems as well as the theory of the operations; all will be considered together with the pertinent basic notions of the fundamental principles, and explained clearly for both *literati* and *vulgari*." Thus, Pacioli's intended readers are both *literati* and *vulgari*—but these latter readers had to have sufficient command of Latin to follow his interjections of a technical word or the occasional sentence in the classical language.

68. Pacioli repeatedly expressed his commitment to bridge theory and practice. In the opening of the *Summa*'s dedicatory letter to Guidobaldo, he stressed the advantages available to those who master theoretical and applied principles of mathematics: "Considering, Your Highness, the immense pleasure and great advantage that the sciences and mathematical disciplines offer to those who know how to apply them to the thinking of specific cases, both theoretical and practical problems, I decided to write the present work . . . primarily for the benefit and pleasure of those who devote themselves to virtuous pursuits." Elsewhere, he clarifies that theoretical principles are valuable insofar as they can be applied to practical cases, as for instance: "However, I am concerned primarily with the application of knowledge, as I mentioned at the beginning; arithmetical and geometric principles and theories will be introduced whenever I will deem it appropriate and necessary" (*Summa,* 12r). His emphasis on application characterizes his overall approach and explains the presence of the text's many illustrations. For example, in the geometry section that concludes the book, Pacioli proposes theoretical problems set in realistic situations, and he illustrates the various cases picturing real objects, from wine caskets and sacks to ladders leaning on walls, to bridge the distance between theoretical principles and their practical applications.

69. The importance of the *Summa* as reference work and introductory text appropriate to prepare students for higher studies is proved by references and even corrections to it that several important mathematicians published, including Girolamo Cardano in the last chapter of his *Practica arithmeticae* (Milan: J. A. Castellioneus, 1539) and Nicolò Tartaglia in his *General trattato di numeri e misure* (Venice: C. Troiano, 1556–60). The best evidence of the preeminence of the *Summa* during the sixteenth century is Federico Commandino's efforts to improve its language and publish the work in a more readable font, a project that he had almost completed when his life was shortened by an untimely death, as Baldi laments (*Le vite de' matematici,* 339).

70. *Summa*'s readers would also know about Pacioli's innovative treatment of accounting techniques—presented "a el modo de Vinegia"—which emphasized the visual dimension. The friar also insisted that one of the three crucial ingredients of the successful businessman was keeping the graphic dimension of accounting books organized neatly so that information could be scanned quickly and retrieved efficiently (198v).

71. The reflection visible on three faces of the polyhedron does not support a precise identification of the building. This reflection fit the received opinion of what a princely—likely Urbino's—palace would look like, and it is integral to the painting's visual rhetoric, connecting its themes and subjects with the Urbinate court, a well-known center of perspective and advanced mathematical studies. On the friar's hypothetical residency in the Marches, see Dante Bernini, "Luca Pacioli alla corte ducale di Urbino," *Antichità Viva* 21 (1982): 36–41.

72. On the cultural background of this polyhedron, see Alberto Pérez-Gómez, "The Glass Architecture of Fra Luca Pacioli," *Architectura* 28 (1998): 156–80. Francesca Cortesi Bosco saw in it a hermetic dimension, claiming also that the crystal depicted was made by one of the members of the Barovier family from Murano, well known for its glassmaking masters. See Bosco, "Il simbolismo ermetico del vetro nel *Fra Luca Pacioli e suo discepolo,*" in *Venezia, Le Marche e la civiltà adriatica,* ed. Ileana Chiappini di Sorio and Laura De Rossi (Venice: Edizioni della Laguna, 2003), 238–41.

73. It is notable that the design with an internal bottom hook and a hole on the top displayed by the painting differs from the solution that Leonardo later chose for hanging the polyhedra discussed by Pacioli in *Divina proportione.*

74. Baldi, *Le vite de' matematici,* 344.

75. The *Summa*'s section "De corporibus regularibus" includes three representations of the dodecahedron (70r). The angle from which the dodecahedron is depicted in the painting differs from those chosen for the illustrations in the *Summa* and *Divina proportione.* A dodecahedron seen from a very similar perspective appears in the intarsia by Luchino Bianchino in the choir of S. Paolo (but now in S. Teresa del Bambin Gesú) in Pavia; the perspective of this intarsia polyhedron seems incorrect. Moreover, the figures of *Divina proportione*'s manuscripts, likely modeled after drawings by Leonardo, present the same string attachment for the dodecahedron. Unlike this and similar intarsia bravuras that aimed at demonstrating their maker's command of perspective and that served as mind games and visual entertainment, our panel's diagrams and polyhedra are part of a structured discourse based on classical and modern texts and are bearers of specific ideas rather than mere visual entertainment. See Massimo Ferretti, "I maestri della prospettiva," in *Storia dell'arte italiana,* ed. Federico Zeri (Turin: Einaudi, 1982), pt. 3, vol. 3, 459–585, esp. 543. On the cultural and artistic dimensions of intarsias, see Bruna Ciati, "Cultura e società nel secondo quattrocento attraverso l'opera ad intarsio di Lorenzo e Cristoforo da Lendinara," in Emiliani, *La prospettiva rinascimentale,* 201–14.

76. The same skills are necessary to work through book 13 of the *Elements.* The kind of visual reasoning skills and geometric or, better, stereometric knowledge that are involved in the dodecahedron's case exemplify the difference between this mathematical looking and seeing and the barrel-gauging skill described by Michael Baxandall in *Painting and Experience in Fifteenth Century Italy* (Oxford: Clarendon Press, 1972), esp. 86–93. Piero's writings instantiate the difference between these two modes of visual thinking: if barrel gauging is a skill that readers learn in *De abaco* as a standard feature of the basic education in the later Middle Ages, polyhedron "gauging" can be learned for the first time in Piero's *De quinque corporibus regularibus libellus* (the contents of which were exploited by Pacioli in the last section of *Divina proportione*), a novel treatise that reflected the recovery of the ancient tradition of Euclidean mathematics. For Piero's treatise, see *Trattato d'abaco: Dal codice ashburnhamiano 280 (359*–219*) della Biblioteca Medicea Laurenziana di Firenze,* ed. Gino Arrighi (Pisa: Domus Galileiana, 1970).

77. At the beginning of Pacioli's biography, Baldi, *Le vite de' matematici,* 331, writes, "Brother Luca deserves to be considered one of the most excellent mathematicians whose lives are described here, because in his time he was a most apt interpreter [*diligentissimo illustratore*] of these disciplines, and for this he was highly regarded by everybody."

78. Held at the National Gallery of Art, Washington, D.C. (October 12, 1991–January 12, 1992), this blockbuster exhibition attracted over 560,000 people. The background of the dust jacket of *Circa 1492: Art in the Age of Exploration* reproduces sections of two pages of an Aztec manuscript. Against this colorful background is set the Capodimonte painting, and their contrast seems to underscore the different conception of space representation and the relative intellectual complexity of their contents (and respective cultures): in Whiggish terms, one mathematical and perspectival, the other colorfully flat.

79. Galileo repeats the idea that the Book of Nature is written in geometric characters in several instances, but most famously at the beginning of *Il Saggiatore.* See Galileo Galilei, *Opere,* ed. Antonio Favaro, 20 vols. (Florence: Barbera, 1909), vol. 6, 232, and vol. 18, 295.

80. The role of images in early modern science has recently become the subject of intense study. Illuminating discussions may be found in Brian Baigrie, ed., *Picturing Knowledge: Historical and Philosophical Problems Concerning the Use of Art in Science* (Toronto: University of Toronto Press, 1996); Wolfang Lefevre et al., eds., *The Power of Images in Early Modern Science* (Basel: Birkhauser, 2003); and Lorraine Daston and Peter Galison, *Objectivity* (New York: Zone Books, 2007). For a historiographic overview of the subject, see Renzo Baldasso, "The Figures of the Scientific Revolution: A Historiographic Inquiry," *Centaurus* 48 (2006): 69–88.